Mechanics - Important Ideas: Collisions (4 pages; 22/4/21)

(1) A perfectly 'elastic' object (where $e = 1$) is one that, on impact with a particular surface, converts all its kinetic energy into elastic potential energy, which is then converted back into the original amount of kinetic energy; ie kinetic energy is conserved.

e is a measure of the relative bounciness of the two materials involved in the collision: the bigger e is, the more the objects will bounce off each other

(2) The loss of kinetic energy will be greatest when $e = 0$.

(3) Whilst energy is generally lost in a collision (as heat & sound), momentum is conserved because total change in momentum = total impulse = 0, since the objects (A and B, say) exert equal and opposite impulses, by Newton's 3rd Law: $F_A = -F_B \Rightarrow F_A t = -F_B$t

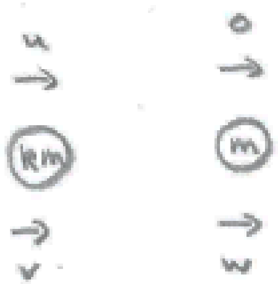
(4) It might seem strange that there is no reference to mass in the Newton's Law of Impact: e = $\frac{v_s}{v_a}$. However, it is involved indirectly, since v_s will be determined by Conservation of Momentum.

(5) The coefficient of restitution between a ball and a surface can be measured as follows (where h_1 is the height that the ball is dropped from, and h_2 is the height that it rises to):

$$e^2 = \frac{\frac{1}{2}mv^2}{\frac{1}{2}mu^2} = \frac{mgh_2}{mgh_1} \text{ (by Conservation of Mechanical Energy)}$$

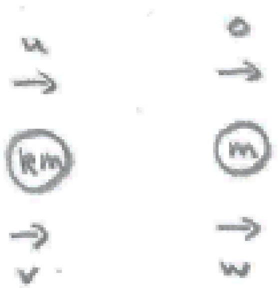
$$= \frac{h_2}{h_1}$$

(6) Referring to the diagram below (where A & B have masses km & m), if $k = 1$ then A cannot reverse its direction, and $v = 0$ only when $e = 1$.



(7) Conditions for A to reverse its direction when A and B collide (A & B have masses km & m)[See separate note for details.]

(I) A has speed u and B is stationary

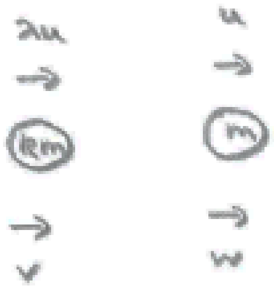


(a) It can be shown that the direction of A is reversed when $e > k$ (whatever value u has).

(b) So if $k \geq 1$, a change of direction isn't possible.

(c) If $k < 1$, a change of direction will be possible provided e is sufficiently big. Note that a bigger e means that A and B bounce off each other more.

(II) A and B are moving in the same direction; A has speed λu ($\lambda > 1$) and B has speed u.

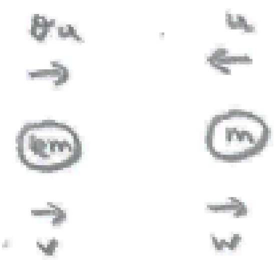


(a) It can be shown that A will never reverse direction if $\frac{\lambda k+1}{\lambda-1} \geq 1$

(b) In particular, A will never reverse direction if $k \geq 1$ or $\lambda \leq 2$

(c) If $k < 1$ & $\lambda > 2$, then A will reverse direction if certain further conditions apply to k & λ, provided that e is big enough.

(III) A and B are moving in opposite directions; A has speed θu and B has speed u.



(a) It can be shown that the condition $k < \frac{\theta+2}{\theta}$ (and sufficiently big e) is necessary and sufficient for A to reverse direction.

(b) In particular, if $k \leq 1$, then A will reverse direction, for sufficiently big e.

(c) And if $k \geq 3$, then A will never reverse direction.