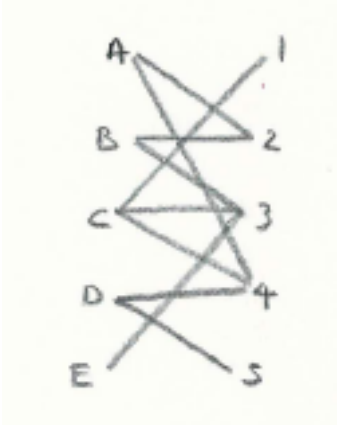
Maximum Matching Algorithm (6/9/2013)

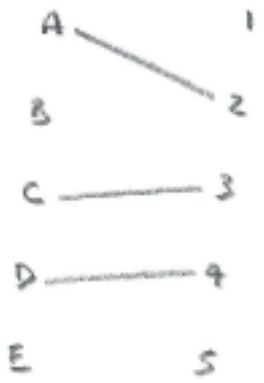
Example



The above diagram shows the possible (ie allowable) allocations of people (A-E) to tasks (1-5). The object of the exercise is to match as many people to tasks as possible. (Each person has one task, and no tasks are shared.)

The Maximum Matching algorithm starts with an 'incomplete matching' (ie where not everyone is matched to a task), and aims to improve it by increasing the number of people matched by one.

For example, we might start with the following 'initial matching':



(Note that these matchings are ones that are allowable, according to the first diagram.)

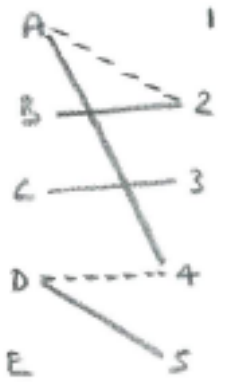
The procedure is then as follows:

Choose a person who hasn't been matched yet; for example, B.

Ideally we would like to be able to match this person to a task that isn't in the initial matching (ie a task that isn't already matched to somebody), and we would then have achieved our aim. However, this would be too easy (and would already have been spotted when the initial matching was created).

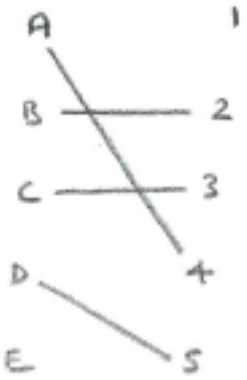
Hence we have no choice but to match B to another task; for example, 2 (note that this has to be one of the allowable matchings specified at the start).

The following diagram should be referred to in the procedure described below.



What we intend to do is to match B to 2 (indicated by a complete line) and remove the existing matching of A to 2 (indicated by a dotted line). This means that we will need to find an alternative match for A.

In this example, it is possible to match A to 4 instead (complete line). At present, 4 is matched to D (dotted line), and so an alternative match must be found for D. Since D will match to 5 (complete line), and 5 is currently free, we have succeeded in finding an improved matching:



The alternating complete and dotted lines are not a standard convention (though you could use them, if you indicated what you were doing).

However, the following notation involving '-' and '=' signs is fairly standard:

The steps in arriving at the improved matching can be summarised as:

$B - 2 = A - 4 = D - 5$ (*)

where the '-' sign represents a new matching that is being set up, and the '=' sign represents an existing matching that is being removed.

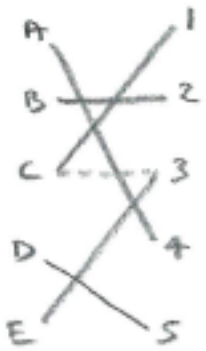
Once the new matching has been set up, (*) would be replaced by

$B = 2 - A = 4 - D = 5$

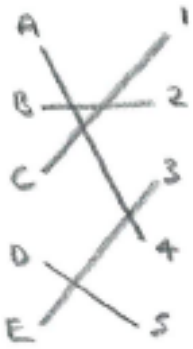
This is the 'alternating path' that the textbooks often refer to.

We can continue from here, to try to find a 'complete matching' (ie one where each person has been matched to a task):

The following diagram should be referred to in the procedure described below.



E can only be matched to 3 (complete line); 3 is currently matched to C (dotted line), and so an alternative matching will need to be found for C. As 1 is currently free (complete line), we have succeeded in finding a complete matching:



The effect of these additional steps can be summarised as $E = 3 - C = 1$