Matrices: Transformations – Exercises

(2 pages ; 3/6/26)

(1) The point P is transformed by the matrix $\begin{pmatrix} 1 & 1 & 2 \\ -2 & 3 & 0 \\ 0 & 4 & -1 \end{pmatrix}$ to the image point (4, 2, –3). Find the coordinates of P.

[without using a calculator]

(2) $M = \begin{pmatrix} a & c \\ b & d \end{pmatrix}$ represents a transformation.

(i) Under what conditions will $x = 0$ be an invariant line?

(ii) Under what conditions will there be an invariant line of the form $x = \lambda$ (where $\lambda \neq 0$)?

(3)(i) Plot the image of the unit square under the transformation represented by the matrix $\begin{pmatrix} 5 & 1 \\ 2 & 3 \end{pmatrix}$

(ii) Use (i), but with more general labels, to show that the area scale factor for the transformation $\begin{pmatrix} a & c \\ b & d \end{pmatrix}$ is $ad - bc$.

(4)(i) Find the equation of the line that the matrix $\begin{pmatrix} 1 & 2 \\ 3 & 6 \end{pmatrix}$ maps all points to.

(ii) For the same transformation, find the equation of the line that maps to the point with an x-coordinate of w.

(iii) For the same transformation, for which point(s) will the

x-coordinate remain unchanged by the transformation?

(5) Derive a formula for the area of a triangle with corners at $(0,0), (a, b), (c, d)$, using matrix transformations.

(6) Show that the matrix $\begin{pmatrix} cos2\theta & sin2\theta \\ sin2\theta & -cos2\theta \end{pmatrix}$ [representing a reflection in the line $y = tan\theta . x$] can be written as $\begin{pmatrix} \frac{1-m^2}{1+m^2} & \frac{2m}{1+m^2} \\ \frac{2m}{1+m^2} & \frac{m^2-1}{1+m^2} \end{pmatrix}$

, where $m = tan\theta$

(7)(i) Show that the transformation represented by the matrix $\begin{pmatrix} 3 & 6 \\ 1 & 2 \end{pmatrix}$ (with determinant zero) maps all points to a particular line.

(ii) Find the line whose points all map to the point $(3,1)$.

(iii) Without doing any calculations, what can be said about the line whose points all map to the point $(6,2)$?

(iv) Write down the line whose points all map to the Origin.