

Matrices: Simultaneous Equations – Exercises

(2 pages ; 3/6/26)

(1) Use matrices to find the plane containing the points $(2, -1, 4)$, $(-3, 4, 2)$ and $(1, 0, 5)$ (without using a calculator)

(2)(i) Three planes are represented by the following equations:

$$x - y + z = 1$$

$$2x + ky + 2z = 3$$

$$x + 3y + 3z = 5$$

For what value of k do the planes not meet at a single point? For this value of k how are the planes configured?

(ii) If $k = 2$, find the point of intersection, using matrices.

(3) Find the value of k for which the following equations are consistent.

$$3x - 3y - z = k$$

$$2x - y - z = 5$$

$$x + 4y - 2z = 7$$

(4) Show that the following three planes meet in a line, giving the equation of that line in cartesian form.

$$x - y + 3z = 4$$

$$4x + 5y - 2z = 8$$

$$x + 17y - 25z = -12$$

(5) Consider the planes with the following equations:

$$\begin{aligned}ax - y + z &= 1 \\2y - z &= b \\4x + 3y - 2z &= 2\end{aligned}$$

(i) Find conditions on a and b for:

- (a) the 3 planes to meet at a single point
- (b) the 3 planes to meet in a line
- (c) no point of intersection of the 3 planes

(ii) Show that in case (c) the line of intersection of the 1st two planes is parallel to the 3rd plane.