

Matrices: Shears – Exercises (1 page; 4/6/26)

(1) Consider the matrix $M = \begin{pmatrix} a & c \\ b & d \end{pmatrix}$, which represents a shear. Show that it is not possible for all of the elements of the matrix to be positive. [It can be assumed that $\text{tr}M = 2$.]

(2) If the 2×2 matrix M represents a shear, what can be said about M^{-1} ? [It can be assumed that the trace of a 2×2 matrix will equal 2 in the case of a shear.]

(3) Find the invariant lines of the shear represented by the matrix $\begin{pmatrix} 4 & -3 \\ 3 & -2 \end{pmatrix}$

(4) Find the invariant lines of the shear represented by the matrix $\begin{pmatrix} 7 & -4 \\ 9 & -5 \end{pmatrix}$

(5) Show that $\frac{(a-1)^2+c^2}{c} = -\frac{b^2+(1-d)^2}{b}$ for the shear $\begin{pmatrix} a & c \\ b & d \end{pmatrix}$.

[It can be assumed that the trace of a 2×2 matrix will equal 2 in the case of a shear.]