

Matrices – Determinants: Exercises (1 page ; 3/6/26)

(1) If $M = \begin{pmatrix} \lambda & k \\ 1 & \lambda - k \end{pmatrix}$, where λ & k are real numbers, what is the range of possible values of k , in order that $|M| > 0$ for all values of λ ?

(2) Factorise the determinant $\begin{vmatrix} x^2 - x & y^2 - y & z^2 - z \\ x & y & z \\ 1 & 1 & 1 \end{vmatrix}$

(3) Write the determinant $\begin{vmatrix} 1 & x^2 & x^4 \\ 1 & y^2 & y^4 \\ 1 & z^2 & z^4 \end{vmatrix}$ as a product of linear factors.

(4) Given that a 3×3 determinant can always be reduced to triangular form (in the same way as simultaneous equations), to

produce a multiple of $\begin{vmatrix} 1 & a & b \\ 0 & 1 & c \\ 0 & 0 & 1 \end{vmatrix}$, show that it can be further

reduced to a multiple of the Identity matrix. [Obviously this is an academic exercise, as the determinant can be evaluated as soon as triangular form has been reached.]