

Maclaurin Series - Exercises (2 pages; 22/3/20)

Key to difficulty:

* easier

** moderate

*** harder

(1***) Find the 1st 3 non-zero terms of the Maclaurin expansions of the following functions, and the intervals of validity of the infinite series:

(i) $\ln(3 - 2x)$

(ii) $\ln\left(\frac{\sqrt{1+3x}}{1-2x}\right)$

(iii) $e^{\cos x}$

(2*) Find a Maclaurin expansion (with 3 non-zero terms) for $\sin^2 x$ by two methods

(3*) Expand \sqrt{x} in powers of $x - 1$, and investigate the accuracy of the resulting approximation for $\sqrt{2}$ when 8 terms are taken.

(4***) Find the % errors (to 2sf) associated with the following Maclaurin approximations:

(i)(a) $\cos x = 1 - \frac{x^2}{2!}$ for $x = \frac{\pi}{6}$ & $x = \frac{\pi}{3}$

(b) $\cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!}$ for $x = \frac{\pi}{6}$ & $x = \frac{\pi}{3}$

(ii)(a) $e^x = 1 + x + \frac{x^2}{2!}$ for $x = 1$ & $x = 2$

$$(b) e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \frac{x^4}{4!} \text{ for } x = 1 \text{ \& } x = 2$$

$$(iii) \ln(1 + x) = x - \frac{1}{2} x^2 \quad \text{for } x = 0.1, x = 0.5 \text{ \& } x = 1$$

$$(b) \ln(1 + x) = x - \frac{1}{2} x^2 + \frac{1}{3} x^3 - \frac{1}{4} x^4$$

$$\text{for } x = 0.1, x = 0.5 \text{ \& } x = 1$$

(5***) Use the 1st 5 terms of a Maclaurin expansion to find an approximate value for $P(Z < 1)$, where $Z \sim N(0,1)$ and Z has pdf

$$f(z) = \frac{1}{\sqrt{2\pi}} \exp\left(-\frac{1}{2} z^2\right)$$

(6***) Use 3 terms of a Maclaurin expansion of $\ln\left(\frac{1+x}{1-x}\right)$ to find an approximate value for $\ln\left(\frac{2}{3}\right)$

(7***) Find the first 3 non-zero terms, as well as the general term in the Maclaurin expansion of $\cosh^3 x$