MAT Exercises - General - Sol'ns (4 pages; 3/11/22)
(1) (i) Does $\sqrt{4}$ equal 2 or $\pm 2$ ? (ii) Simplify $\sqrt{x^{2}}$

Solution
(i) $\sqrt{4}=2$
[By convention, the square root symbol denotes the positive root (consider the $\pm$ in the quadratic formula, which wouldn't be needed if the square root symbol covered both positive and negative roots). Note that the solution of $x^{2}=4$ is $x= \pm \sqrt{4}$.]
(ii) $\sqrt{x^{2}}=|x|$ [Note that $x$ could be negative.]
(2) Find the square roots of $49-12 \sqrt{5}$

Solution
Let $x^{2}=49-12 \sqrt{5}$
Consider $x=a+b \sqrt{5}$
Then $a^{2}+2 a b \sqrt{5}+5 b^{2}=49-12 \sqrt{5}$
Let $a^{2}+5 b^{2}=49 \& 2 a b=-12$
[a variation on Equating Coefficients]
Looking for integer solutions, we see that either $a=2 \& b=-3$ or $a=-2 \& b=3$ work.

