MAT Exercises – General - Sol'ns (4 pages; 3/11/22)

(1) (i) Does $\sqrt{4}$ equal 2 or ± 2 ? (ii) Simplify $\sqrt{x^2}$

Solution

(i)
$$\sqrt{4} = 2$$

[By convention, the square root symbol denotes the positive root (consider the \pm in the quadratic formula, which wouldn't be needed if the square root symbol covered both positive and negative roots). Note that the solution of $x^2 = 4$ is $x = \pm \sqrt{4}$.

(ii)
$$\sqrt{x^2} = |x|$$
 [Note that x could be negative.]

(2) Find the square roots of $49 - 12\sqrt{5}$

Solution

Let
$$x^2 = 49 - 12\sqrt{5}$$

Consider
$$x = a + b\sqrt{5}$$

Then
$$a^2 + 2ab\sqrt{5} + 5b^2 = 49 - 12\sqrt{5}$$

Let
$$a^2 + 5b^2 = 49 \& 2ab = -12$$

[a variation on Equating Coefficients]

Looking for integer solutions, we see that either

$$a = 2 \& b = -3 \text{ or } a = -2 \& b = 3 \text{ work.}$$