MAT - Exam Technique (12 pages; 29/9/21)

Contents

(A) Introduction

Although the syllabus for the MAT is limited to roughly the 1st year of A Level, you may be able to gain an advantage from additional knowledge (especially in the Multiple Choice section, where any method can be used). See “MAT – Extra Material”.

Don't do anything that is too obscure: the correct approach, once found, is usually relatively 'simple'. Always consider the simplest possible interpretation of anything that is unclear about the question.

If a topic looks unfamiliar, remember that knowledge outside the syllabus is not assumed, so the question should be self-contained and include definitions of new concepts. Usually such questions turn out to be easier than normal, as the candidate is being rewarded for coping with an unfamiliar topic. Typically the first part (for the Long Questions) will turn out to be quite simple.

Whatever exam techniques you adopt, it is probably best to allow them to develop gradually over the course of the preparation period, so that by the time you come to the exam the techniques will have been tested.

(B) Multiple Choice questions

For the earlier questions in the Multiple Choice section, some answers are easy to eliminate, and it is not unusual to be able to arrive at the correct answer entirely by a process of elimination (though obviously it would be a good check to establish the validity of the correct answer independently).

Sometimes the correct answer can be deduced from its form only (eg if it has to be of the form $a \leq k \leq b$).

However, there is obviously a danger that time may be spent eliminating some of the answers, only to find that a direct approach is needed in order to decide between the remaining options.

As a general principle, look for (useful) things that are quick to do (ie where you can quickly establish whether they are leading anywhere).

Often a decision will need to be made as to whether an algebraic or graphical approach is appropriate. Bear in mind that a simplifying feature of a problem might only be revealed once a diagram has been drawn.

The Multiple Choice questions tend to cover the same themes each year; generally:

(1) Largest/smallest of various expressions

(2) Maximum/minimum value of an expression

(3) Matching a graph to a function

(4) Existence of solutions

(5) Number of solutions

See the separate document "MAT - Index of Questions”, which also lists recommended practice questions.

By the way, it has been suggested that Admissions tutors may read 'rough' work for the Multiple Choice section!

(C) Long questions

The long questions are also fairly similar from year to year (though the topics involved are harder to prepare for). Typically they are as follows (2010 however was an exception):

Q2: relations between functions; iterative relations

Q3: graphs of polynomials

Q4: areas

Q5: robot / sequences

(D) Approaches & Ideas

(D1) Observations

(1) Identification of simplifying features

(a) When solving $f(x) = g(x)$, it may be the case that $f(x)$ attains a maximum when $g(x)$ attains a minimum. (This device has been used more than once in MAT questions.)

(b) If n is an integer, it may be the case that a solution only exists for even n , for example.

(2) Do the Multiple Choice options give any clues? It may help you to get on the wavelength of the question. For the Long Questions, look ahead for ideas.

(3) If the first part of a question seems very easy, it is highly likely to be needed for the next part.

(4) Ensure that all of the information provided in the question has been used.

(5) If a unique solution is required, or if there is to be exactly 2 solutions, or no solutions, then this may suggest the solving of a quadratic equation and considering the discriminant $b^2 - 4ac$.

A condition in the form of an inequality can also suggest the use of $b^2 - 4ac$.

(6) If you are told that $x \neq a$, then the solution may involve a division by $x - a$.

(7) The presence of a $\pm$ sign often suggests that a square root is being taken at some stage (eg to solve a quadratic equation).

(D2) Creating equations

(1) Equations can be created from:

(a) information in the question

(b) relevant definitions and theorems

(Look out for standard prompts to use a particular result. For example, a reference to a tangent to a circle can suggest the result that the radius and the tangent are perpendicular.)

If necessary, create your own variables (for example, a particular length in a diagram).

Sometimes the advantage of creating an equation is that it gives you something to manipulate; ie in order to make further progress.

(2) When setting up equations or inequalities:

(i) use k^2 to represent a positive number

(ii) use $2k$ to represent an even number; $2k+1$ to represent an odd number

(D3) Case by case approach

(1) Example: Solve $\frac{x^2+1}{x^2-1} < 1$

Case 1: $x^2-1<0$; Case 2: $x^2-1>0$

(Once we know whether x^2-1 is positive or negative, we can multiply both sides of the inequality by it, changing the direction of the inequality as necessary.)

(2) Transitional (or 'critical') points

This involves considering the point(s) at which the nature of a problem changes.

Example 1: $\frac{(x-1)(x+2)(x-3)}{(x+1)(x-2)(x+3)} < 0$

The only points at which the sign of the left-hand side can change are at the roots of $(x-1)(x+2)(x-3)=0$, and at the vertical asymptotes $x=-1, x=2$ and $x=-3$

Example 2: If investigating a situation involving the intersection of a curve and a straight line, perhaps consider first the case where the line touches the curve (ie is a tangent).

(D4) Reformulating a problem

(1) For example, in order to solve the equation $f(x)=k$, consider where the graph of $y=f(x)$ crosses the line $y=k$.

Or turn an equation into the intersection of a curve and a straight line.

Or, more generally, rearrange an equation to $f(x) = g(x)$, and find where the curves $y = f(x)$ & $y = g(x)$ meet (or show that they won't meet).

(2) To sketch the cubic $y = x^3 + 2x^2 + x + 3$, rewrite it as

$y = x(x^2 + 2x + 1) + 3$, and translate the graph of

$y = x(x^2 + 2x + 1) = x(x + 1)^2$

(3) Pairs of numbers x, y might be represented by the coordinates (x, y); eg to find possible integer values of x & y, determine the grid points within the relevant area.

(4) To show that a function $f(n)$ of an integer n cannot be a perfect square, perhaps show instead that $f(n) - 1$ is always a perfect square.

(5) To show that two functions $f(n)$ & $g(n)$ cannot be equal, perhaps show that they belong to different classes; eg even and odd numbers.

(6) Make a substitution. As a simple example, writing $y = 2^x$ might turn an equation into a quadratic.

(D5) Experimenting

(1) Often inspiration for a particular problem will only come after experimenting (including drawing a diagram); or an important feature of a problem will not become apparent until then.

(2) Examples

(i) Draw a diagram

This may reveal a hidden feature of the problem; eg a triangle may turn out to be right-angled, or the solution may lie on the boundary of a region.

(ii) Try out particular values (eg $n = 1$)

This may reveal a simplifying feature of the problem (eg if an integer n is involved, then perhaps it has to be even).

(iii) Consider what happens when $x \to \infty$.

(iv) Use approximate values to get a feel for a problem (eg $log_{10}3 \approx \frac{1}{2}$); especially for comparing Multiple Choice answers.

(v) Consider a simpler version of the problem (eg experiment with a simple function such as $y = x^2$).

(vi) Start to list the terms of a sequence - a pattern may emerge.

For counting problems, find a systematic way of listing the possibilities, and then of counting the items in the list.

(E) Common Pitfalls

(1) Not using a specified method. Even if a superior method is used instead, zero marks are usually awarded for not using the method requested in the question.

(2) Using $\Rightarrow$ when $\Leftrightarrow$ is required.

(3) Losing a solution of an equation by dividing out a factor.

(4) Multiplying an inequality by a quantity without realising that it is (or could be) negative (eg $\ln\left(\frac{1}{2}\right)$).

(5) Beware of not considering all cases, or not giving special treatment to certain cases (eg where there would otherwise be division by zero).

(6) Not justifying an argument fully (eg " $a^3 < b^3 \Leftrightarrow a < b$ " probably needs "because $y = x^3$ is an increasing function").

(F) Presentation of work

(1) Avoid writing out a result (eg at the start of a question) without making it clear that it is still to be proved.

(2) Suppose a candidate has written the following:

$xy < xz$

$y < z$

$x > 0$

As it stands, it is not clear whether $y < z$ is supposed to lead on from $xy < xz$ (which would be incorrect as it stands, as x could be negative), or whether $y < z$ is a result established earlier (or perhaps stated in the question). Likewise, is $x > 0$ being deduced, or brought in from somewhere else?

A revised version of the above might be:

From (*), $xy < xz$

Also, the question states that $y < z$

Hence $x > 0$

(3) To avoid any uncertainty, each statement in your working needs something to show where it comes from. Often this will just be an implies sign ($\Rightarrow$) (but see the discussion below about the "if and only if" argument).

Introducing something with the word "consider" is a good way to indicate that you are starting a new line of argument.

(4) For “if and only if” proofs, it may be acceptable to indicate that the line of reasoning is reversible (assuming that this is the case, of course); ie by use of the $\Leftrightarrow$ sign at each stage.

When proving that $A = B$, be careful not to adopt the following (incorrect) argument:

"$A = B \Rightarrow \cdots \Rightarrow Y = Z$ (eg $0 = 0$) [something that is clearly true]"

What we want to show instead is that

"$Y = Z$ [which is clearly true] $\Rightarrow \cdots \Rightarrow A = B$

This can be got round by writing

"$Y = Z \Leftrightarrow \cdots \Leftrightarrow A = B$

(though it isn't usually thought to be that elegant, and is best avoided if possible).

A more acceptable approach is to show that $A = X$ and that $B = Y$, and then demonstrate that X can be rearranged into Y; or alternatively show that $X - Y = 0$

(5) Explanations

It can be a good idea to explain what you are doing, for the marker's benefit. (However, credit won't be given for an explanation of what you would do, if you had more time.)

Ensure that arguments are fully justified, and that the order of steps is clear (rather than spread all over the page). It should be clear where each step comes from.

(6) Showing working

Reasons for showing plenty of working:

(a) It enables you to easily check over your work (as you complete each line).

(b) The marker is kept happy, by not forcing them to do work in their head.

(c) If you make a slip, then your method may still be clear (bearing in mind that method marks are usually available).

(d) Full marks may not be awarded in the case of a 'show that' result if there is a jump to the result.

(G) Checking

(1) Read over each line before moving on to the next one. This is the most efficient way of picking up any errors.

(2) Just before embarking on a solution, re-read the question. Also re-read it when you think you have finished answering the question, in case there is an additional task that you have forgotten about. It is also a good idea to re-read the question if you find yourself getting bogged down in awkward algebra, or if you don't seem to be getting anywhere.

(H) Use of time

(1) Before embarking on a solution, consider how likely it is that it will work, and how much time it will take.

(2) Save time by using letters to represent recurring expressions (eg "write $y = sin^2x$").

(3) You might like to save a relatively straightforward task to complete in the last few minutes of the exam, rather than frantically looking through the paper for something to check.