

2011 MAT Paper - Multiple Choice (6 pages; 30/8/20)**Q1/A****Solution**

(a) starts in the wrong quadrant, and so can be eliminated.

$$\text{If } f(x) = x^3 - x^2 - x + 1,$$

$$f'(x) = 3x^2 - 2x - 1 = (x - 1)(3x + 1)$$

[Had $(x - 1)$ not been a factor, (c) could have been eliminated.]

Thus there is a stationary point at $x = 1$,

and so **the answer must be (c)**, by elimination.

Q1/B**Solution**

Let the sides of the rectangle be x & y . Then $P = 2(x + y)$ and $A = xy$.

Of the 4 possibilities, (c) is the first one that looks feasible.

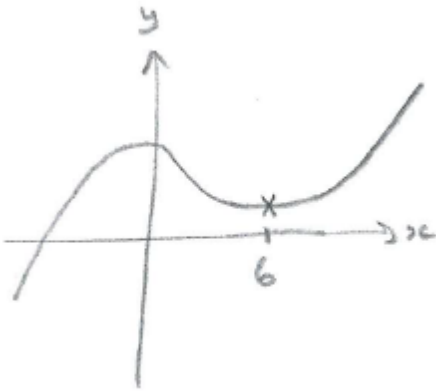
$$\text{Then } P^2 - 16A = 4(x^2 + y^2 + 2xy) - 4(4xy) = 4(x - y)^2 \geq 0$$

So the answer is (c).

Q1/C**Solution**

$$\text{Let } y = x^3 - 9x^2 + 631.$$

$$\text{Then } \frac{dy}{dx} = 0 \Rightarrow 3x^2 - 18x = 0 \Rightarrow x = 0 \text{ or } 3x - 18 = 0; \text{ ie } x = 6$$



Thus, $x = 5$ is the largest integer, for which $y(x) > y(x + 1)$.

So the answer is (a).

Notes

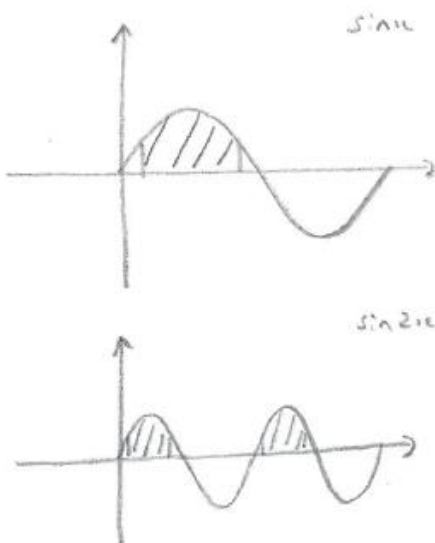
The official sol'ns are missing a " $>$ " sign; ie $-3n^2 + 15n + 8 > 0$

Using the 'official' method, to be absolutely sure that $n = 5$ is the largest value, we can set $\frac{15+\sqrt{A}}{6} = 5$; which gives $A = 225$, so that

$\frac{15+\sqrt{321}}{6} > 5$ (and already shown to be < 5.5), whilst $\frac{15-\sqrt{321}}{6} < 5$

Q1/D

Solution



One or both of the inequalities is true for the following regions:

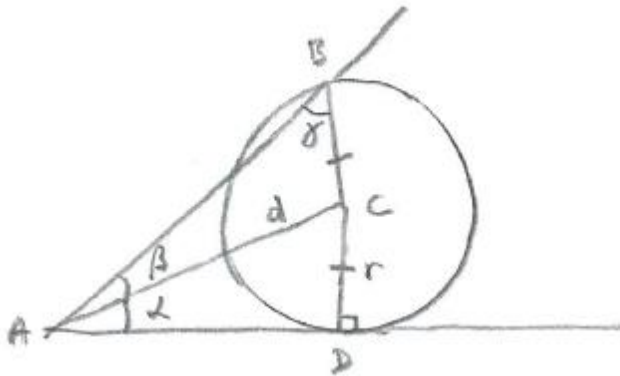
$$\left[\frac{\pi}{12}, \frac{5\pi}{6}\right] \& \left[\pi + \frac{\pi}{12}, \pi + \frac{5\pi}{12}\right]$$

So the required proportion is $\frac{\left(\frac{(9+4)\pi}{12}\right)}{2\pi} = \frac{13}{24}$

So the answer is (b).

Q1/E

Solution



Drawing in the radius CD, as in the diagram,

$$\sin\alpha = \frac{r}{d} \& \frac{\sin\beta}{r} = \frac{\sin\gamma}{d}$$

$$\text{so that } \sin\alpha = \frac{\sin\beta}{\sin\gamma} \text{ or } \sin\beta = \sin\alpha \sin\gamma$$

So the answer is (b).

Q1/F

Solution

$$x^2 + y^2 + 4x\cos\theta + 8y\sin\theta + 10 = 0$$

$$\Rightarrow (x + 2\cos\theta)^2 - 4\cos^2\theta + (y + 4\sin\theta)^2 - 16\sin^2\theta + 10 = 0$$

$$\begin{aligned} \text{Then, for a circle of radius } r, r^2 &= 4\cos^2\theta + 16\sin^2\theta - 10 \\ &= 4 + 12\sin^2\theta - 10 \\ &= 12\sin^2\theta - 6 \end{aligned}$$

$$\text{And } 12\sin^2\theta - 6 > 0 \Rightarrow \sin^2\theta > \frac{1}{2},$$

$$\text{so that, for } 0 \leq \theta < \pi, \sin\theta > \frac{1}{\sqrt{2}}$$

$$\Rightarrow \frac{\pi}{4} < \theta < \frac{3\pi}{4}$$

So the answer is (b).

Q1/G

Solution

$$\text{Note that, for } -1 \leq x \leq 1, 0 \leq x^2 \leq 1$$

$$\text{so that } -1 \leq x^2 - 1 \leq 0$$

So $f(x^2 - 1) = (x^2 - 1) + 1$ (since the equation of the left-hand sloping part of the graph is $y = x + 1$)

$$\text{and hence } \int_{-1}^1 f(x^2 - 1)dx = \int_{-1}^1 x^2 dx = \left[\frac{1}{3}x^3 \right]_{-1}^1 = \frac{2}{3}$$

So the answer is (d).

Q1/H

Solution

$$\begin{aligned} x &= 8^{\log_2 x} - 9^{\log_3 x} - 4^{\log_2 x} + \log_{0.5} 0.25 \\ &= 2^{3\log_2 x} - 3^{2\log_3 x} - 2^{2\log_2 x} + 2 \\ &= (2^{\log_2 x})^3 - (3^{\log_3 x})^2 - (2^{\log_2 x})^2 + 2 \end{aligned}$$

$$\Rightarrow x = x^3 - x^2 - x^2 + 2$$

$$\Rightarrow x^3 - 2x^2 - x + 2 = 0$$

[Unsurprisingly,] $x = 1$ is a root,

$$\text{giving } (x - 1)(x^2 - x - 2) = 0,$$

$$\text{so that } (x - 1)(x + 1)(x - 2) = 0$$

and there are two positive values of x .

So the answer is (c).

Q1/I

Solution

$$\text{Let } y = \sin^2 x, \text{ so that } y^4 + (1 - y)^3 = 1$$

$$\Rightarrow y^4 - y^3 + 3y^2 - 3y = 0$$

$$\Rightarrow y(y^3 - y^2 + 3y - 3) = 0$$

$$\Rightarrow y(y - 1)(y^2 + 3) = 0$$

$$\Rightarrow \sin^2 x = 0 \text{ or } 1$$

$$\Rightarrow x = 0, \pi, \frac{\pi}{2} \text{ or } \frac{3\pi}{2}$$

So the answer is (b).

Q1/J

Solution

$$f(1) = 1$$

$$f(2) = f(1) = 1$$

$$f(3) = [f(1)]^2 - 2 = -1$$

$$f(4) = f(2) = 1$$

$$f(5) = [f(2)]^2 - 2 = -1$$

$$f(6) = f(3) = -1$$

$$f(7) = [f(3)]^2 - 2 = -1$$

$$f(8) = f(4) = 1$$

Thus $f(n) = -1$ when n is odd (except for $n = 1$).

For even n , $f(n) = 1$ if n is a power of 2,

and $f(n) = -1$ if n is of the form $(2^k)(2p + 1)$ (ie all other even numbers)

Of the 100 values being added,

$$n = 1 \Rightarrow 1 \text{ [1 value]}$$

$$\text{other odd} \Rightarrow -1 \text{ [49 values]}$$

$$\text{powers of 2} \Rightarrow 1 \text{ [6 values]}$$

$$\text{other even numbers} \Rightarrow -1 \text{ [44 values]}$$

$$\text{So } f(1) + f(2) + f(3) + \dots + f(100)$$

$$= (1 + 6) - (49 + 44) = -86$$

So the answer is (a).