# 2007 MAT - Multiple Choice (6 Pages; 27/8/20)

## Q1/A

### Introduction

A useful question to ask is: "What can we easily do?" In this case, we can break down the expression into powers of 2 and 3.

It's only when we simplify the expression as much as possible that it becomes apparent that the problem is easily solved.

### Solution

$$\frac{6^{r+s}\times 12^{r-s}}{8^r\times 9^{r+2s}} = \frac{2^{(r+s)+2(r-s)}\times 3^{(r+s)+(r-s)}}{2^{3r}\times 3^{2(r+2s)}}$$

$$= \frac{2^{3r-s}\times 3^{2r}}{2^{3r}\times 3^{2r+4s}}$$

$$= \frac{1}{2^s\times 3^{4s}}$$

For this to be an integer, we require $s \le 0$.

**So the answer is (b).**

## Q1/B

### Solution

$f(x)$ is maximised when $3sin^2(10x+11) = 0$, and $f(x) = 49$

**So the answer is (c).**

## Q1/C

### Solution

$7sinx + 2cos^2x = 5$

$\Rightarrow 7sinx + 2(1 - sin^2x) = 5$

$\Rightarrow 2sin^2x - 7sinx + 3 = 0$

$\Rightarrow sinx = \frac{7 \pm \sqrt{25}}{4} = 3$ (reject) or $\frac{1}{2}$

For $0 \leq x \leq 2\pi$, there are 2 sol'ns.

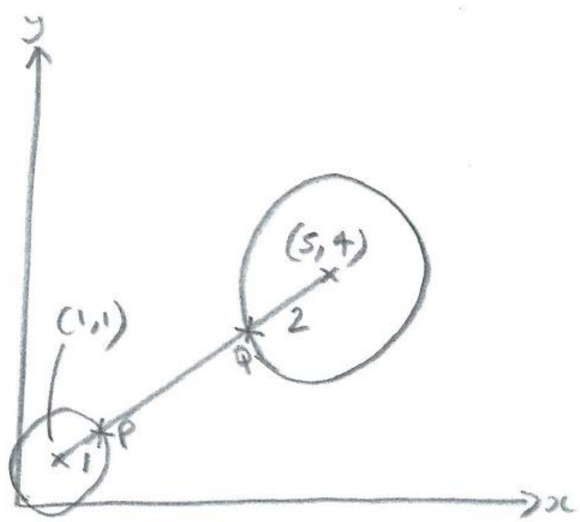
**So the answer is (b).**

**Q1/D**

**Introduction**

Drawing a diagram may indicate how to proceed with the problem.

**Solution**




From the diagram, we see that the required point must lie on the line joining the centres of the two circles. (If a different point is selected, then it is possible to reduce the distance PQ by placing P and Q on the line joining the centres.)

[It is possible to find the intersection of the large circle with the line joining the centres, but this is quite time-consuming (especially for a multiple choice question), and so it's worth looking for another method.]

[The official solution uses a vector approach. A variation of this approach, which doesn't use vectors, is to use linear interpolation.]

The distance between the two centres is 5 (from the Pythagorean triple $(3,4,5)$), and the required point is $\frac{2}{5}$ of the way along the line joining the centres, from the point (5,4).

Taking a weighted average of the two centres [' linear interpolation']:

$\frac{2}{5}(1,1)+\frac{3}{5}(5,4)=(\frac{17}{5},\frac{14}{5})$ or $(3.4, 2.8)$

**So the answer is (a).**

**Q1/E**

**Solution**

Write $f(x,n)=(1-x)^n(2-x)^{2n}(3-x)^{3n}(4-x)^{4n}(5-x)^{5n}$

If $x=1$, then $f(x,n)=0$, so that (a) and (d) are false.

With $x>5$, if $n$ is odd, sign of $f(x,n)$ is $(-)(+)(-)(+)(-)\Rightarrow -$

**So the answer is (b).**

[As a check, with $x>5$, if $n$ is even, sign of $f(x,n)$ is $(+)(+)(+)(+)(+)\Rightarrow +$]

**Q1/F**

**Solution**

Let $y=2^x$

Then $8^x + 4 = 4^x + 2^{x+2}$

$\Rightarrow y^3 + 4 = y^2 + 4y$

or $f(y) = y^3 - y^2 - 4y + 4 = 0$

Now, $f(1) = 0$, so that $y - 1$ is a factor of $f(y)$,

and $f(y) = (y - 1)(y^2 - 4)$,

so that $2^x = 1, 2$ or $-2$ (reject)

This gives rise to 2 real sol'ns for $x$ (0 and 1).

**So the answer is (c).**

**Q1/G**

**Solution**

Let $f(x) = 2^{-x} sin^2(x^2)$

As $f(0) = 0$, (d) can be eliminated.

Also, as $f(0) \geq 0$, (b) can be eliminated.

The presence of $x^2$ indicates that the peaks of the graph will not occur at regular intervals, and so (c) can be eliminated.

**So the answer is (a).**

**Q1/H**

**Solution**

By considering the integrals as areas under the curve, the equations can be converted into simultaneous equations in two unknowns: $A = \int_0^1 f(x)dx$ & $B = \int_1^2 f(x)dx$ , with the required answer being $A + B$.

Thus $3A + 2B = 7$ and $(A + B) + B = 1$

so that $2B = 7 - 3A$ & $2B = 1 - A$

Hence $7 - 3A = 1 - A$, and $A = 3$; $B = -1$,

so that $A + B = 2$

**So the answer is (d).**

## Q1/I

### Introduction

This is an example of a question that perhaps looks more complicated than it actually is.

### Solution

$a$ is maximised when $(log_{10}b)^2$ is minimised; ie when $b = 1$, giving $4(log_{10}a)^2 = 1$,

so that $log_{10}a = \frac{1}{2}$ (rejecting $-\frac{1}{2}$, as $a$ is not maximised).

Thus $a = 10^{\frac{1}{2}}$ or $\sqrt{10}$

## Q1/J

### Solution

Consider $n = 1$ [as this is fairly quick to do]

The LHS is $100 + \frac{1}{2}(100)(101) = 5150$

As the LHS increases with $n$, the inequality will hold provided that $k < 5150$

**So the answer is (d).**

## Conclusion

This is a rather strange question, in that hardly any work is involved (especially for question J, which is often the hardest), if you happen to consider $n = 1$ straightaway.