Logarithms (4 pages; 13/2/17)

(1) Converting an exponential equation into a logarithmic equation (definition of the logarithm).

Example 1: $2^x = 9$

The equivalent logarithmic equation takes the form $log_a b = c$

where a is the base (2) and c is the power (x).

This is how the logarithm is defined, and it can be seen to be the power in the exponential equation.

So $log_2 9 = x$

Obviously the reverse process could be carried out:

Example 2: $log_2 8 = x$

The base is 2 and the power is x.

So $2^x = 8$, and hence $x = 3$ (the exponential equation only helps because 8 is a simple power of 2).

(2) Logarithm function as the inverse of the exponential function

If $y = a^x$, then $log_a y = x$

Thus, x is mapped to y by the rule $y = a^x$,

and y is mapped to x by the rule $x = log_a y$

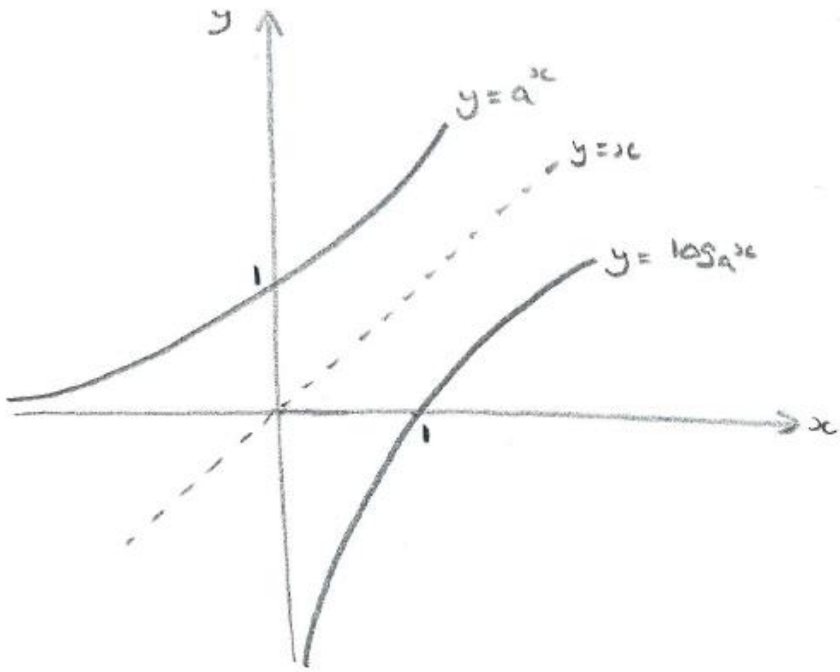
Then, in order for this to be expressed as a function with x (rather than y) on the horizontal axis, we write $y = log_a x$

Thus the logarithm function is the opposite of the exponential function, and vice versa. Each is the 'inverse function' of the other.

[Note: $y = a^x$ is referred to here as the (general) exponential function, but "**the** exponential function" is often taken to mean $y = e^x$, with other exponential functions described as "**an** exponential function".]

The graphs of inverse functions are obtained from each other by reflection in the line $y = x$. In this case, $y = a^x$ and $x = log_a y$ represent the same graph (as we have only made a rearrangement of the equation), but by converting $x = log_a y$

to $y = log_a x$ we are swapping the roles of x & y, which is equivalent to a reflection in the line $y = x$. See diagram below.



(3) 'Cancelling'

As the logarithmic function is the inverse of the exponential function, and vice versa, the operations of raising to a power and taking the logarithm can be thought of as cancelling each other out, so that $log_a(a^x) = x$ and $a^{log_a x} = x$

(4) Taking the log of both sides

Example: A population is modelled by the equation $P = a.2^{bt}$

Option 1: Take logs to base 2

$log_2P = log_2a + log_2(2^{bt}) = log_2a + bt$

(so that if log_2P is plotted against t, a straight line pattern should emerge, if the model is correct; the line has a gradient of b and

'y-intercept' of log_2a)

Option 2: Take logs to base 10 (eg if logs to base 2 can't be calculated)

$log_{10}P = log_{10}a + log_{10}(2^{bt}) = log_{10}a + bt.log_{10}2$

(which, again, is the equation of a straight line).

(5) Change of base

Approach 1

Example: $(10^2)^3 = 10^6$

Let $a = 10, b = 10^2$ & $x = 10^6$

Then $log_ab\,.log_bx = 2 \times 3 = 6 = log_ax$

Thus $\boldsymbol{log_ab\,.log_bx = log_ax}$

["going from a to b, and then from b to x"]

Alternative Forms:

(a) $log_ab\,.log_bx = log_ax$ can also be interpreted as converting log_bx to log_ax by applying the factor log_ab (though this may not be as memorable), or (more naturally) as converting $log_a x$ to $log_b x$ by applying the factor $log_b a$

(b) Quotient of two logs (to the same base): $\frac{log_a x}{log_a b} = log_b x$

which may be relabelled as $\frac{log_x b}{log_x a} = log_a b$

(c) In (b), when $x = b$: $log_a b = \frac{1}{log_b a}$

Approach 2

The problem is to find an expression for $log_b x$, given $log_a x$

Let $log_a x = m$, in order to be able to create the exponential equation $a^m = x$

Then $log_b(a^m) = log_b x$, so that $mlog_b a = log_b x$

and $log_b x = log_a x . log_b a$