Linear Coding (2 pages; 8/7/21)

To establish the effect on $\bar{x}, s$ & r of the transformation

$y = ax + b$:

$$\bar{y} = \frac{\sum y_i}{n} = \frac{\sum(ax_i + b)}{n} = \frac{(a\sum x_i) + nb}{n} = a\bar{x} + b$$

$s_y{}^2 = \frac{S_{yy}}{n}$, where $S_{yy} = (\sum y_i{}^2) - n\bar{y}^2$

$= (\sum(ax_i + b)^2) - n(a\bar{x} + b)^2$

$= (a^2 \sum x_i{}^2) + (2ab \sum x_i) + nb^2 - na^2\bar{x}^2 - 2nab\bar{x} - nb^2$

$= a^2\{(\sum x_i{}^2) - n\bar{x}^2\}$, since $\sum x_i = n\bar{x}$

$= a^2 S_{xx}$

Hence $s_y{}^2 = \frac{a^2 S_{xx}}{n} = a^2 s_x{}^2$, and so $s_y = as_x$

Suppose that we are considering the correlation between x_i and z_i.

Then $r_{xz} = \frac{S_{xz}}{\sqrt{S_{xx}S_{zz}}}$ and $r_{yz} = \frac{S_{yz}}{\sqrt{S_{yy}S_{zz}}}$

Now $S_{yz} = (\sum y_i z_i) - n\bar{y}\bar{z}$

$= (\sum(ax_i + b)z_i) - n(a\bar{x} + b)\bar{z}$

$= (a\sum x_i z_i) + (b\sum z_i) - na\bar{x}\bar{z} - nb\bar{z}$

$= a\{(\sum x_i z_i) - n\bar{x}\bar{z}\}$, since $\sum z_i = n\bar{z}$

$= aS_{xz}$

Hence $r_{yz} = \frac{aS_{xz}}{\sqrt{a^2 S_{xx} S_{zz}}} = \frac{S_{xz}}{\sqrt{S_{xx} S_{zz}}} = r_{xz}$