Linear Programming – Q9b: Simplex method

[Practice/M](18/6/21)

Maximise $5x - 2y + 4z$, subject to the following constraints:

$2x + y - z \leq 6$

$x - y + 2z \geq 5$

$3x + y - 7z \geq 4$

$x \geq 0, y \geq 0, z \geq 0$

Apply the Big M (Simplex) method, as far as establishing the pivot row for the 2nd time.

Solution

Step 1: (As for the 2 Stage method), create equations with either slack variables, or surplus and artifical variables, as appropriate

$P - 5x + 2y - 4z = 0$ (1)

$2x + y - z + s_1 = 6$ (2)

$x - y + 2z - s_2 + a_1 = 5$ (3)

$3x + y - 7z - s_3 + a_2 = 4$ (4)

Step 2: Modify the objective to maximising $P' = 5x - 2y + 4z - M(a_1 + a_2)$

$= 5x - 2y + 4z - M[(5 - x + y - 2z + s_2) + (4 - 3x - y + 7z + s_3)]$

$= (5 + 4M)x - 2y + (4 - 5M)z - Ms_2 - Ms_3 - 9M$

(where M is a large number)

Step 3: Represent the equations in a Simplex tableau:

P'	x	y	z	s_1	s_2	s_3	a_1	a_2	value	row
1	$-5-4M$	2	$-4+5M$	0	M	M	0	0	$-9M$	(1)
0	2	1	-1	1	0	0	0	0	6	(2)
0	1	-1	2	0	-1	0	1	0	5	(3)
0	(3)	1	-7	0	0	-1	0	1	4	(4)

Step 4: As we are maximising P', we look for large negative coefficients of variables in the 1st row (so that when the variable is maximised, it will increase P' as much as possible). Here we

take x as the pivot column, and perform the ratio test to establish the pivot row.

row 2: $\frac{6}{2} = 3$; row 3: $\frac{5}{1} = 5$; row 4: $\frac{4}{3}$; so the pivot row is row 4 (indicated in the table above by the brackets - or circling if handwritten)

Step 5: Eliminate x from rows 1-3

P'	x	y	z	s_1	s_2	s_3	a_1	a_2	value	row
1	0	$\frac{4M+11}{3}$	$\frac{-13M-47}{3}$	0	M	$\frac{-M-5}{3}$	0	$\frac{5+4M}{3}$	$\frac{-11M+20}{3}$	(5)=(1)+ (5+4M)(8)
0	0	$\frac{1}{3}$	$\frac{11}{3}$	1	0	$\frac{2}{3}$	0	$-\frac{2}{3}$	$\frac{10}{3}$	(6)=(2) −2(8)
0	0	$-\frac{4}{3}$	$(\frac{20}{3})$	0	-1	$\frac{1}{3}$	1	$-\frac{1}{3}$	$\frac{11}{3}$	(7)= (3) − (8)
0	1	$\frac{1}{3}$	$-\frac{7}{3}$	0	0	$-\frac{1}{3}$	0	$\frac{1}{3}$	$\frac{4}{3}$	(8)=(4)÷ 3

Step 6: As the value of P' still involves M, we look for large negative coefficients of variables in the 1st row again, and so take z as the pivot column. Row 8 can be ignored, when establishing the pivot row, due to its negative coefficient of z.

row 6: $\frac{(\frac{10}{3})}{(\frac{11}{3})} = \frac{10}{11}$; row 7: $\frac{(\frac{11}{3})}{(\frac{20}{3})} = \frac{11}{20} < \frac{10}{11}$, so the pivot row is row 7