Linear Programming – Q9a: Simplex method

[Practice/M](18/6/21)

Maximise $5x - 2y + 4z$, subject to the following constraints:

$2x + y - z \leq 6$

$x - y + 2z \geq 5$

$3x + y - 7z \geq 4$

$x \geq 0, y \geq 0, z \geq 0$

Apply the 1st stage of the 2 Stage Simplex method, as far as establishing the pivot row for the 2nd time.

Solution

Step 1: Create equations with either slack variables, or surplus and artifical variables, as appropriate.

$P - 5x + 2y - 4z = 0$ (1)

$2x + y - z + s_1 = 6$ (2)

$x - y + 2z - s_2 + a_1 = 5$ (3)

$3x + y - 7z - s_3 + a_2 = 4$ (4)

Step 2: Let $A = a_1 + a_2 = (5 - x + y - 2z + s_2) + (4 - 3x - y + 7z + s_3)$

$= 9 - 4x + 5z + s_2 + s_3$

The 1st stage of the method is to minimise A.

Step 3: Represent the equations in a Simplex tableau:

A	P	x	y	z	s_1	s_2	s_3	a_1	a_2	value	row
1	0	4	0	-5	0	-1	-1	0	0	9	(1)
0	1	-5	2	-4	0	0	0	0	0	0	(2)
0	0	2	1	-1	1	0	0	0	0	6	(3)
0	0	1	-1	2	0	-1	0	1	0	5	(4)
0	0	(3)	1	-7	0	0	-1	0	1	4	(5)

Step 4: As we are minimising A, we look for large positive coefficients of variables in the 1st row (so that when the variable

is maximised, it will reduce A as much as possible). Here there is no choice, and we take x as the pivot column, and perform the ratio test to establish the pivot row (ignoring any rows with negative coefficients of x).

row 3: $\frac{6}{2} = 3$; row 4: $\frac{5}{1} = 5$; row 5: $\frac{4}{3}$; so the pivot row is row 5 (indicated in the table above by the brackets - or circling if handwritten)

[Note: It is possible (though less usual) to maximise $-A$ instead.]

Step 5: Eliminate x from rows 1-4

A	P	x	y	z	s_1	s_2	s_3	a_1	a_2	value	row
1	0	0	$-\frac{4}{3}$	$\frac{13}{3}$	0	-1	$\frac{1}{3}$	0	$-\frac{4}{3}$	$\frac{11}{3}$	(6)=(1)−4(10)
0	1	0	$\frac{11}{3}$	$-\frac{47}{3}$	0	0	$-\frac{5}{3}$	0	$\frac{5}{3}$	$\frac{20}{3}$	(7)=(2)+5(10)
0	0	0	$\frac{1}{3}$	$\frac{11}{3}$	1	0	$\frac{2}{3}$	0	$-\frac{2}{3}$	$\frac{10}{3}$	(8)=(3) −2(10)
0	0	0	$-\frac{4}{3}$	$(\frac{13}{3})$	0	-1	$\frac{1}{3}$	1	$-\frac{1}{3}$	$\frac{11}{3}$	(9)=(4) −(10)
0	0	1	$\frac{1}{3}$	$-\frac{7}{3}$	0	0	$-\frac{1}{3}$	0	$\frac{1}{3}$	$\frac{4}{3}$	(10)=(5)÷ 3

Step 6: As A hasn't yet been reduced to zero, we look for large positive coefficients of variables in the 1st row again, and so take z as the pivot column. Rows 7 and 10 can be ignored, when establishing the pivot row, due to their negative coefficients of z.

row 8: $\frac{(\frac{10}{3})}{(\frac{11}{3})} = \frac{10}{11}$; row 9: $\frac{(\frac{11}{3})}{(\frac{13}{3})} = \frac{11}{13} < \frac{10}{11}$, so the pivot row is row 9