Linear Programming – Q8: Simplex method

[Practice/M](18/6/21)

Minimise $-3x + 2y + z$, subject to the following constraints:

$x + y - 4z \leq 4$

$-x + 3y + 2z \geq -2$

$x \geq 0, y \geq 0, z \geq 0$

Use the ordinary Simplex method to solve this problem.

Solution

Step 1: Rewrite the problem as

Maximise $P = 3x - 2y - z$,

subject to $x + y - 4z \leq 4$ and $x - 3y - 2z \leq 2$

Step 2: Create equations with slack variables [it is possible to skip this step, and go straight to the Simplex tableau]:

$P - 3x + 2y + z = 0$ (1)

$x + y - 4z + s_1 = 4$ (2)

$x - 3y - 2z + s_2 = 2$ (3)

Step 3: Represent the equations in a Simplex tableau:

P	x	y	z	s_1	s_2	value	row
1	-3	2	1	0	0	0	(1)
0	1	1	-4	1	0	4	(2)
0	(1)	-3	-2	0	1	2	(3)

Step 4: Choose x as the pivot column (as it has the largest negative coefficient in the objective row), and perform the ratio test to establish the pivot row.

As $\frac{2}{1} < \frac{4}{1}$, row 3 is the pivot row (indicated in the table above by the brackets - or circling if handwritten).

Step 5: Eliminate x from rows 1 and 2

As the coefficient of x for row 3 is already 1, no adjustment is needed for that row.

P	x	y	z	s_1	s_2	value	row
1	0	-7	-5	0	3	6	(4)=(1)+3(6)
0	0	(4)	-2	1	-1	2	(5)=(2)-(6)
0	1	-3	-2	0	1	2	(6)=(3)

Step 6: y now has the largest negative coefficient in the objective row, and as the coefficient of y in row 6 is negative, we can take row 5 as the pivot row.

Step 7: Eliminate y from rows 4 and 6

As the coefficient of y for row 5 is 4, we need to divide that row by 4 first.

P	x	y	z	s_1	s_2	value	row
1	0	0	$-8\frac{1}{2}$	$1\frac{3}{4}$	$1\frac{1}{4}$	$9\frac{1}{2}$	(7)=(4)+7(8)
0	0	1	$-\frac{1}{2}$	$\frac{1}{4}$	$-\frac{1}{4}$	$\frac{1}{2}$	(8)=(5)÷ 4
0	1	0	$-3\frac{1}{2}$	$\frac{3}{4}$	$\frac{1}{4}$	$3\frac{1}{2}$	(9)=(6)+3(8)

Step 8: Although z has a negative coefficient in the objective row, the other coefficients of z are negative, and so no further progress can be made.

Hence the solution is: $x = 3\frac{1}{2}, y = \frac{1}{2}, z = 0, s_1 = 0, s_2 = 0, P = 9\frac{1}{2}$, and hence the minimised value of $-3x + 2y + z$ is $-9\frac{1}{2}$

[Check: $x + y - 4z = 4 \leq 4$ and $-x + 3y + 2z = -2 \geq -2$]