

Linear Programming – Q5: Formulating as LP problem

[5 marks](15/6/21)

Exam Boards

OCR : -

MEI: MWA

AQA: -

Edx: -

Workers A-E are to be allocated tasks, so that each worker carries out one task, and each task is carried out by one worker. The table below shows the tasks that each worker is trained to do. The aim is to match up workers to tasks in such a way that as many workers as possible are occupied. Formulate this as a linear programming problem.

	1	2	3	4	5
A		Y	Y		Y
B	Y	Y			
C		Y		Y	Y
D	Y		Y		
E		Y		Y	

[5 marks]

Solution

With A_2, A_3, \dots, E_4 being binary variables, where $A_2 = 1$ means that worker A carries out task 2:

Maximise $P = A_2 + A_3 + A_5 + B_1 + B_2 + C_2 + C_4 + C_5 + D_1 + D_3 + E_2 + E_4$ [1 mark]

[ie maximise the number of matchings]

subject to the following constraints:

$A_2 + A_3 + A_5 \leq 1$ [ie at most one of A_2, A_3, A_5 can be 1] [1 mark]

$B_1 + B_2 \leq 1$

$C_2 + C_4 + C_5 \leq 1$

$D_1 + D_3 \leq 1$

$E_2 + E_4 \leq 1$ [1 mark]

$B_1 + D_1 \leq 1$ [ie at most one of B_1, D_1 can be 1] [1 mark]

$A_2 + B_2 + C_2 + E_2 \leq 1$

$A_3 + D_3 \leq 1$

$C_4 + E_4 \leq 1$

$A_5 + C_5 \leq 1$ [1 mark]