Linear Programming – Q4: Formulating as LP problem

[9 marks](15/6/21)

Exam Boards

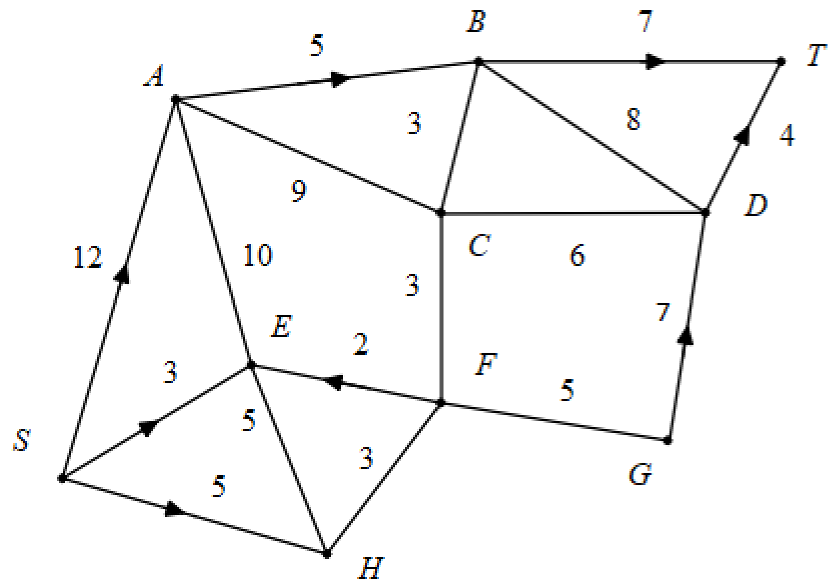
OCR : -

MEI: MwA

AQA: -

Edx: -

The network below shows the maximum capacity for each arc of a network. It is required to maximise the flow across the network, from S to T. Formulate this as a linear programming problem.



[9 marks]

Solution

With SA, AB etc being non-negative integers, representing the flows along the arcs:

Maximise $P = \text{SA} + \text{SE} + \text{SH}$, [1 mark]

subject to the following constraints:

$SA \le 12, SE \le 3, SH \le 5$
$AB \le 5, AC \le 9, CA \le 9, AE \le 10, EA \le 10$
$BC \le 3, CB \le 3, BD \le 8, DB \le 8, BT \le 7$
$CD \le 6, DC \le 6, CF \le 3, FC \le 3$
$DT \le 4, GD \le 7$
$FE \le 2, EH \le 5, HE \le 5$
$FG \le 5, GF \le 5, FH \le 3, HF \le 3$ [4 marks]

Inflows must equal outflows:
At A: $SA + EA + CA = AE + AC + AB$
At B: $AB + CB + DB = BT + BD + BC$
At C: $AC + BC + DC + FC = CA + CB + CD + CF$
At D: $BD + CD + GD = DB + DC + DT$
At E: $SE + AE + FE + HE = EA + EH$
At F: $CF + GF + HF = FE + FC + FG + FH$
At G: $FG = GF + GD$
At H: $SH + EH + FH = HE + HF$

[4 marks]

[The inflow to T will automatically equal the outflow from S.]