Linear Programming – Q3: Formulating as LP problem [6 marks](15/6/21)
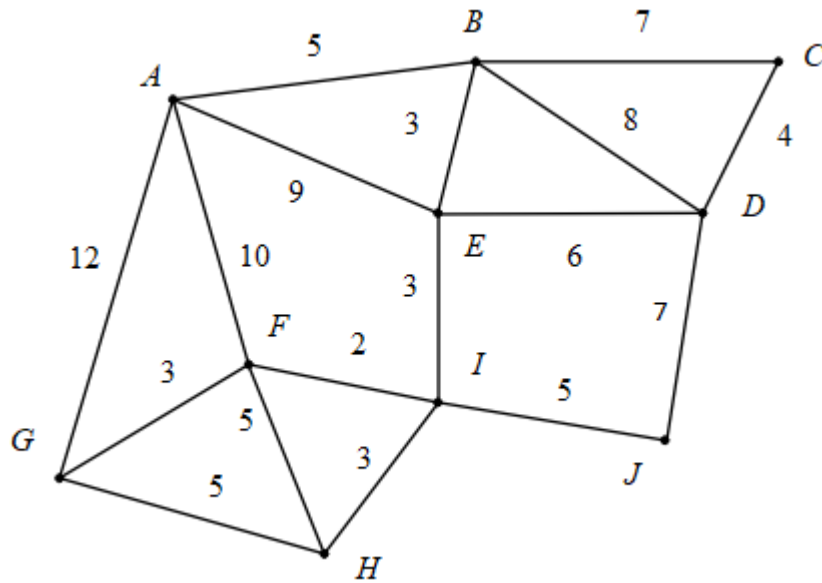
Exam Boards

OCR : -

MEI: MwA

AQA: -

Edx: -

It is required to find the shortest distance between A and J in the network below. Formulate this as a linear programming problem.

[6 marks]

Solution

With AB, AE etc being binary variables, where $AB = 1$ means that the arc AB is travelled along:

Minimise $P = 5AB + 9AE + 10AF + 12AG + 7BC + 7CB + 8BD + 8DB + 3BE + 3EB + 4CD + 4DC + 6DE + 6ED + 7DJ + 3EI + 3IE + 3FG + 3GF + 2FI + 2IF + 5FH + 5HF + 5GH + 5HG + 3HI + 3IH + 5IJ$

[2 marks]

[Arcs not involving A or J can be travelled along in either direction, and so are duplicated.]

$AB + AE + AF + AG = 1$ [the path has to pass along just one of the arcs leading from A]

[1 mark]

$DJ + IJ = 1$ [the path has to pass along just one of the arcs leading to J]

$AB + EB + DB + CB = BE + BD + BC$ [1 mark]

[if we enter B, then we must leave it - each side will total either 0 or 1]

$BC + DC = CB + CD$ [similarly for C]

$BD + CD + ED = DB + DC + DE + DJ$ [D]

$AE + BE + DE + IE = EB + ED + EI$ [E]

$AF + GF + HF + IF = FG + FH + FI$ [F]

$AG + FG + HG = GF + GH$ [G]

3

$FH + GH + IH = HF + HG + HI$ [H]

$EI + FI + HI = IE + IF + IH + IJ$ [I] [2 marks]