

# Linear Programming – Q2c [14 marks](18/6/21)

## Exam Boards

OCR : D (Year 2)

MEI: MwA

AQA: D (Year 2)

Edx: D1 (Year 2)

The following Linear Programming problem is to be solved:

$$\text{Minimise } P = 3x + 2y,$$

$$\text{subject to } 5x + 3y \geq 20$$

$$y \leq 3x$$

$$x \geq 0, y \geq 1$$

Apply the Big M Simplex method, up to the point where the 1st pivot has been completed, and the 2nd is about to be carried out.

[14 marks]

## Solution

We wish to maximise  $-P = -3x - 2y$  [1 mark]

The constraint equations are:

$$5x + 3y - s_1 + a_1 = 20$$

$$y - 3x + s_2 = 0$$

$$y - s_3 + a_2 = 1$$

$$x, s_1, s_2, s_3 \geq 0 \quad [3 \text{ marks}]$$

$$\text{Let } P' = -P - M(a_1 + a_2)$$

$$= -3x - 2y - M(20 - 5x - 3y + s_1) - M(1 - y + s_3)$$

$$\text{so that } P' + x(3 - 5M) + y(2 - 4M) + Ms_1 + Ms_3 = -21M$$

[3 marks]

The initial Simplex tableau is

basic variable	$x$	$y$	$s_1$	$s_2$	$s_3$	$a_1$	$a_2$	value
$a_1$	5	3	-1	0	0	1	0	20
$s_2$	-3	1	0	1	0	0	0	0
$a_2$	0	1	0	0	-1	0	1	1
$P'$	$3 - 5M$	$2 - 4M$	M	0	M	0	0	$-21M$

[4 marks]

Take  $x$  as the pivot column [as  $3 - 5M < 2 - 4M$ ] [1 mark]

basic variable	$x$	$y$	$s_1$	$s_2$	$s_3$	$a_1$	$a_2$	value	ratio
$a_1$	5	3	-1	0	0	1	0	20	4
$s_2$	-3	1	0	1	0	0	0	0	n/a
$a_2$	0	1	0	0	-1	0	1	1	n/a
$P'$	$3 - 5M$	$2 - 4M$	M	0	M	0	0	$-21M$	n/a

Applying the ratio test, the pivot row is found to be the 1st one.

[1 mark]

Making the coeff. of  $x$  equal to 1:

basic variable	$x$	$y$	$s_1$	$s_2$	$s_3$	$a_1$	$a_2$	value	row
$a_1$	1	0.6	-0.2	0	0	0.2	0	4	1
$s_2$	-3	1	0	1	0	0	0	0	2
$a_2$	0	1	0	0	-1	0	1	1	3
$P'$	3 - 5M	2 - 4M	M	0	M	0	0	-21M	4

[1 mark]

Remove  $x$  from rows 2, 3 & 4:

$$R2' = R2 + 3(R1'), R3' = R3, R4' = R4 + (5M - 3)(R1')$$

basic variable	$x$	$y$	$s_1$	$s_2$	$s_3$	$a_1$	$a_2$	value	row
$x$	1	0.6	-0.2	0	0	0.2	0	4	1'
$s_2$	0	2.8	-0.6	1	0	0.6	0	12	2'
$a_2$	0	1	0	0	-1	0	1	1	3'
$P'$	0	0.2 - M	0.6	0	M	M - 0.6	0	-M - 12	4'

[3 marks]