Linear Programming – Q2b [13 marks](18/6/21)

Exam Boards

OCR : D (Year 1)

MEI: MwA

AQA: -

Edx: D1 (Year 1)

The following Linear Programming problem is to be solved:

Minimise $P = 3x + 2y$,

subject to $5x + 3y \geq 20$

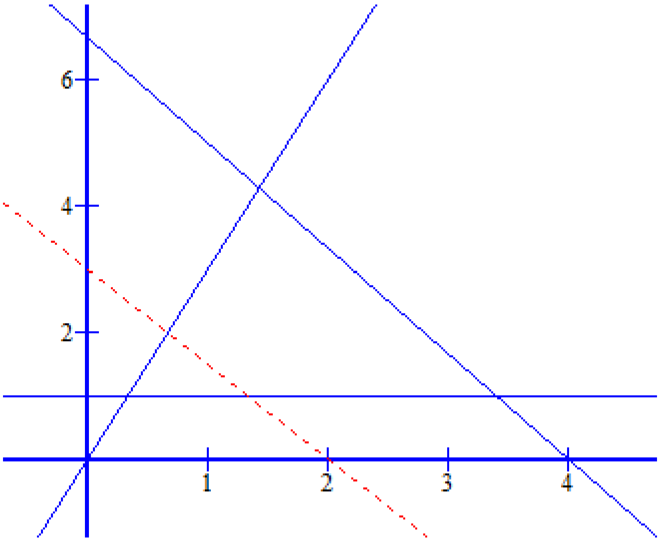
$$y \leq 3x$$
$$x \geq 0, y \geq 1$$

(i) Obtain a solution using a graphical approach. Assume that non-integer solutions are acceptable. [6 marks]

(ii) Obtain a good integer solution. [7 marks]

Solution

(i) The diagram shows the constraint lines, as well as the (dotted) line $3x + 2y = 6$, which is parallel to the objective function.



[3 marks]

As the line representing the objective function moves away from the Origin, it first enters the feasible region at the intersection of $5x + 3y = 20$ and $y = 1$; ie at the vertex $(3\frac{2}{5}, 1)$, when

$P = 3(3.4) + 2(1) = 12.2$ [3 marks]

(ii) To establish an integer solution, consider first of all $x = 3$:

Then $P = 3x + 2y$, and $5x + 3y \geq 20$, $y \leq 3x$ & $y \geq 1$

So $3y \geq 5$; ie $y \geq \frac{5}{3}$, and $y \leq 9$, and $y \geq 1$, [2 marks]

so that $y = 2$ minimises $P = 3x + 2y$, with $P = 13$. [2 marks]

Then consider $x = 4$, which gives $3y \geq 0$ and $y \leq 12$ & $y \geq 1$
[1 mark]

so that $y = 1$ minimises $P = 3x + 2y$, with $P = 14$. [1 mark]

Thus the required integer solution is $(3,2)$, with $P = 13$.

[1 mark]

[Strictly speaking, this is just a good integer solution, as it might be the case that the best integer solution lies elsewhere in the feasible region.]