Kinematics (7 pages; 7/8/17)

Introduction

When we consider the motion of an object, we are interested in its location at a particular time, as well as its velocity and acceleration.

Displacement is the location of an object relative to a chosen point, which need not be the Origin (and need not be the starting point of the object, when $t = 0$). It is displacement that is used predominantly in Kinematics.

Position and distance will also appear, and the distinction between these various terms will be discussed shortly.

Velocity is the rate of change of displacement and acceleration is the rate of change of velocity.

We will see that a force is required in order to create an acceleration.

Kinematics is the branch of Mechanics that concerns the relations between displacements, velocities and accelerations, without considering the forces involved.

Statics and Dynamics are the branches of Mechanics that concern forces. In the case of Statics, there is no motion, and hence the forces are balanced. Dynamics covers the more general case where there is motion (though the forces could also be balanced if there is no acceleration). Objects will often be treated as though they are 'particles'; i.e. having no size. This means that we don't have to worry about any possible rotation of the object.

Units

There are three basic units that are relevant to Mechanics: length, time and mass. Other quantities such as velocity, acceleration and force are combinations of these.

The internationally accepted measures for these units - the S.I. units (Système International d'Unités) - are the metre (m), the second (s), and the kilogram (kg).

Scalars and vectors

As we have already seen, quantities that have a direction associated with them, as well as a magnitude, are referred to as vectors; as opposed to scalars which only have a magnitude.

A displacement is a vector: even if motion is in a single dimension, we distinguish between a displacement of, say, $2m$ and $-2m$.

In two dimensions, we have displacements such as

$3\underline{i} + 4\underline{j} \quad m$

Velocity is also a vector, and is treated in the same way. So, in one dimension, we might have a velocity of either $2ms^{-1}$ (read as "2 metres per second" and sometimes written as $2m/s$) or $-2ms^{-1}$. The speed (a scalar quantity) would be $2ms^{-1}$ in both cases. In two dimensions, the velocity might be $3\underline{i} + 4\underline{j} \quad ms^{-1}$. The speed in this case would be the length (or magnitude) of the vector; ie $5ms^{-1}$ (by Pythagoras).

Displacement, position and distance

In some situations, the term 'distance' is used. Depending on the context, this may be simply the magnitude of the displacement, so that for a displacement of $3\underline{i} + 4\underline{j} \quad m$, the object is at a distance of $5m$ from the chosen reference point (i.e. where the displacement is $0\underline{i} + 0\underline{j} \quad m$).

To consider an alternative use of the term distance, suppose that an object is moving in one dimension, and that its direction is reversed at some point (so if its velocity is positive initially, it becomes negative). For example, the object may move to the right for $5m$, and then reverse its direction, travelling for another $7m$. If its initial displacement is $10m$ (say), then its final displacement will be $8m$, and the distance travelled is $12m$.

Another complication concerns the use of the term 'position'. In Kinematics, position is always measured relative to the Origin. So in the above example, the initial displacement is $10m$, but this could be a position of say $15m$ (to the right of the Origin).

Instantaneous velocity, average velocity and average speed

If the velocity of an object is constant, say $2ms^{-1}$, and its initial displacement is $4m$, for example, then the displacement after 1 second will be $6m$, and the displacement after 2 seconds will be $8m$.

If the velocity is not constant, then we can talk about an **instantaneous velocity**, which is the gradient of the displacement, where the displacement is a function of time.

Note the following definitions:

Average velocity $= \frac{change\ in\ displacement}{time\ taken}$

Average speed $= \frac{distance\ travelled}{time\ taken}$

Thus, in the one-dimensional example mentioned earlier, where the object moves to the right for $5m$, and then to the left for $7m$, if the time taken is $4s$, the average velocity is $\frac{-2}{4} = -\frac{1}{2}ms^{-1}$, whereas the average speed is

$\frac{12}{4} = 3ms^{-1}$,

Acceleration

An acceleration of e.g. $3ms^{-2}$ (read as "3 metres per second squared" and sometimes written as $3m/s^2$) means that the velocity is increasing by $3ms^{-1}$ every second. In the case of constant acceleration, if the initial velocity is $2ms^{-1}$, then the velocity after 1 second will be $5ms^{-1}$, and

the velocity after 2 seconds will be $8ms^{-1}$, and so on.

If the acceleration is not constant, then we can talk about an instantaneous acceleration, which is the gradient of the velocity, where the velocity is a function of time.

Displacement-time graphs

When motion is in one dimension (i.e. takes place on a line) we can create a graph of displacement against time, where the gradient of the displacement is the

instantaneous velocity.

Figure 1

Figure 1 shows the progress of a cyclist, who initially accelerates from rest, at a constant rate of $2ms^{-2}$, thus reaching a velocity of $10ms^{-1}$ after $5s$. At this point he continues at a constant velocity for $10s$, before decelerating at a constant rate of $1ms^{-2}$, to come to rest again after another $10s$.

The acceleration cannot readily be found from the displacement-time graph. The velocity-time graph is needed for this.

Velocity-time graphs

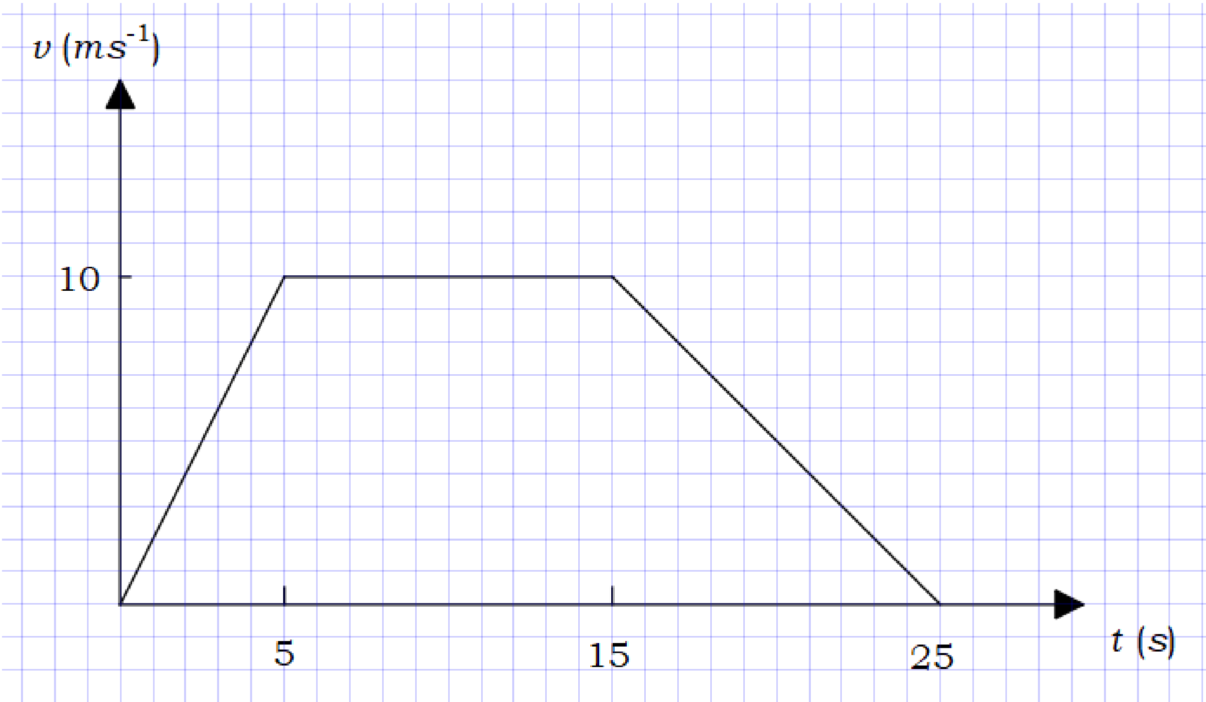

Figure 2

Figure 2 shows the graph of velocity against time for the same situation. In the first $5s$, the gradient of the velocity (which is the acceleration) is $2ms^{-2}$, during the next $10s$ it is zero, and during the final $10s$ it is $-1ms^{-2}$.

The velocity-time graph also provides information about the distance travelled: this is the area under the graph.

Thus, over the first $5s$, the average velocity is

$\frac{1}{2}(0+10)=5\ ms^{-1}$, and the distance travelled is

average velocity $\times$ time $=5\times 5=25m$

(Note that this formula is only valid when the velocity goes up at a constant rate; i.e. when the acceleration is constant.)

Exercise: Find the total distance travelled by the cyclist.

Solution

Over the next $10s$ the distance travelled

$=$ velocity $\times$ time $=10\times 10=100m$

Over the final $10s$, the average velocity is

$\frac{1}{2}(10+0)=5\ ms^{-1}$, and so the distance travelled is

$5\times 10=50m$

Thus the total distance travelled is

$25+100+50=175m$

An important point to note is that, if the object reverses direction at any point (so that its velocity becomes negative), we will need to determine the area under the time axis separately. We will find the distance travelled in the positive direction, and then add to that the distance travelled in the negative direction.

If we were to use integration to find areas, then any region under the time axis would give a negative area. Supposing that the area above the time axis is A, and that the area below the time axis is

$-B$ (where A and B are positive). Then the total distance travelled is $A + B$, whilst $A - B$ would be the final displacement.

Note: In the above example, it has been assumed that the cyclist can instantaneously change from having no acceleration to having e.g. an acceleration of $2ms^{-2}$; i.e. the velocity-time graph has sharp corners to it. In practice of course the change would be gradual, and the 'corners' would be rounded.

Motion with constant acceleration

See the separate note on the 'suvat' equations.