Kinematics – Q1 [Practice/M] (7/6/21)

Given that the velocity of a particle as a function of its displacement is $v(x) = 10e^{-x}$ and that $x = 0$ when $t = 0$, find:

(i) the acceleration as a function of x

(ii) x as a function of t

(iii) v as a function of t

(iv) the acceleration as a function of t

Given that the velocity of a particle as a function of its displacement is $v(x) = 10e^{-x}$ and that $x = 0$ when $t = 0$, find:

(i) the acceleration as a function of x

(ii) x as a function of t

(iii) v as a function of t

(iv) the acceleration as a function of t

Solution

(i) $a(x) = \frac{dv}{dt} = \frac{dv}{dx}.\frac{dx}{dt} = v\frac{dv}{dx} = 10e^{-x}(-10e^{-x}) = -100e^{-2x}$

[The relation $a(x) = \frac{d}{dx}\left(\frac{1}{2}v^2\right)$ is sometimes more convenient to use.]

(ii) $\frac{dx}{dt} = 10e^{-x}$

$\Rightarrow \int e^x dx = 10\int dt$

$\Rightarrow e^x = 10t + C$

$x = 0$ when $t = 0 \Rightarrow C = 1$

$\Rightarrow x(t) = \ln(10t + 1)$

(iii) From (ii), $v(t) = 10e^{-\ln(10t+1)}$

$\Rightarrow v(t) = 10(10t + 1)^{-1}$

(iv) From (i) & (ii), $a(t) = -100e^{-2\ln(10t+1)}$

$\Longrightarrow a(t) = -100(10t + 1)^{-2}$

Check:

$\frac{d}{dt}x(t) = \frac{1}{10t+1}(10) = 10(10t + 1)^{-1} = v(t)$

$\frac{d}{dt}v(t) = 10(-1)(10t + 1)^{-2}(10)$

$= -100(10t + 1)^{-2} = a(t)$