

Integration Exercises - Part 2 (Harder)(3 pages; 4/7/26)

$$(1) \int \frac{1}{1+e^x} dx$$

$$(2) \int e^{2x}(1+e^x)^{\frac{1}{2}} dx$$

$$(3) \int \frac{1}{\sqrt{x}(1+\sqrt{x})} dx$$

$$(4) \int \sec x dx$$

$$(5) \int \frac{(\ln x)^2}{x} dx$$

$$(6) \int \frac{x^5}{4-x^3} dx$$

$$(7) \int \sqrt{1+\sin 2x} dx$$

$$(8) \int \arctan x dx$$

$$(9) \int \frac{1}{x \ln x} dx$$

$$(10) \int \frac{e^x}{e^{2x}+1} dx$$

$$(11) \int \frac{\sec^2 x}{4+\tan^2 x} dx$$

$$(12) \int \frac{1}{1-\sin x} dx$$

$$(13) \int_{-\frac{3}{2}}^{\frac{3}{2}} \frac{1}{4x^2+9} dx$$

$$(14) \int \frac{x}{1+x^4} dx$$

$$(15) \int \frac{\sin 2x}{1+\cos x} dx$$

$$(16)^* \int_0^1 \sqrt{16x^2+9} dx$$

$$(17)^* \int \sec^3 x dx$$

$$(18) \int \sec^4 x \, dx$$

$$(19) \int \cos^5 x \, dx$$

$$(20) \int \cos x \ln(\cos x) \, dx$$

$$(21) \int \tan x \sin x \, dx$$

$$(22) \int \sin 3x \cos x \, dx$$

$$(23) \int \operatorname{cosec} x \, dx$$

$$(24) \int \tan^3 x \, dx$$

$$(25) \int \tan^4 x \, dx$$

$$(26) \int \frac{\cos^3 x}{\sin^2 x} \, dx$$

$$(27) \int \frac{\cos x + \sin x}{\cos x - \sin x} \, dx$$

$$(28) \int \tan x \sec x \, dx$$

$$(29) \int \arcsin\left(\frac{x}{3}\right) \, dx$$

$$(30)^* \int \frac{\sin x}{\sin x + \cos x} \, dx$$

$$(31) \text{ Find a reduction formula for } I_n = \int_0^1 x^n \sqrt{1-x^2} \, dx$$

$$(32) \int_1^e (\ln x)^2 \, dx$$

$$(33) \int 2^x \, dx$$

$$(34) \int \frac{1}{x^2+6x+18} \, dx$$

$$(35) \int \frac{x^2}{1+x^6} \, dx$$

$$(36) \int \frac{1}{(2x^2+3)^{\frac{3}{2}}} \, dx$$

$$(37)^{**} \int \sqrt{4x^2-1} \, dx$$

$$(38) \int \frac{4x+5}{\sqrt{4-6x-x^2}} dx$$

$$(39) \int \frac{\sin\sqrt{x}}{\sqrt{x}} dx$$

(40) Find a reduction formula for $I_n = \int_0^2 x^n \sqrt{4-x^2} dx$, and hence show that $\int_0^2 x^5 \sqrt{4-x^2} dx = \frac{1024}{105}$

(41) Find a reduction formula for $I_n = \int_0^\pi \cos^n x dx$, and hence show that $\int_0^\pi \cos^4 x dx = \frac{3\pi}{8}$

(42) Use the substitution $t = \tan\left(\frac{x}{2}\right)$ to find $\int_0^{\frac{\pi}{2}} \frac{1}{2+\sin x} dx$

$$(43) \int \operatorname{cosec} x \sec x dx$$

$$(44) \int \operatorname{cosec}^2 x dx$$

$$(45) \int_0^{\frac{\pi}{2}} \sin^4 x dx$$