

Integration – Miscellaneous: Exercises (1 page; 27/3/26)

(1) Find the values of the following integrals, or show that they are not defined.

(i) $\int_{-2}^{\infty} \frac{1}{x^2} dx$

(ii) $\int_{-\infty}^{-\frac{1}{2}} e^{2x} dx$

(iii) $\int_0^1 x^{-\frac{2}{3}} dx$

(2) Explain the following 'paradox':

$$\int \frac{1}{2x} dx = \frac{1}{2} \int \frac{1}{x} dx = \frac{1}{2} \ln x + C$$

but $\int \frac{1}{2x} dx = \frac{1}{2} \ln(2x) + C$ (by the reverse Chain rule)

(3) Given that $\int \frac{1}{x} dx = \ln x$ for $x > 0$, show that $\int \frac{1}{x} dx = \ln|x|$ for all $x \neq 0$