Impulse & Momentum – Q4 (11/6/23)

A spaceship has a geostationary orbit about the earth (ie it stays above the same point on the earth's surface). An astronaut walks from one end of the spaceship to the other. Describe what happens, relative to the earth's surface.

Solution

Let the spaceship, excluding the astronaut, have mass M, and let the astronaut have mass m. Suppose that the astronaut is walking with velocity w relative to the spaceship, and that the spaceship (including the astronaut) travels at velocity v relative to the earth's surface, once the astronaut has started walking.

By conservation of momentum,

$Mv + m(v + w) = 0 \quad \Rightarrow v(M + m) = -mw$

and so $v = -\frac{mw}{(M+m)}$

ie the spaceship moves in the opposite direction to the motion of the astronaut relative to the spaceship.

Consider the motion of the centre of mass of the spaceship and astronaut.

Its velocity relative the the earth's surface is the weighted average of the velocities of the spaceship (excluding the astronaut) and the astronaut:

$\left(\frac{M}{M+m}\right) v + \left(\frac{m}{M+m}\right)(v + w)$

$= \left(\frac{1}{M+m}\right)(Mv + mv + mw)$

$= v + \frac{mw}{M+m} = 0$

As there is no external force on the spaceship and astronaut, we would expect there to be no net motion of the centre of mass of the spaceship and astronaut.