Impulse & Momentum – Q3 (11/6/23)

A ball is projected vertically upwards at a speed of $5ms^{-1}$ when it is $3m$ above the ground. Given that it just returns to its original height after bouncing on the ground (and assuming that there is no air resistance), find the coefficient of restitution between the ball and the ground. (Assume that $g = 9.8$)

Solution

Method 1

Taking downwards to be the positive direction, and applying $'v^2 = u^2 + 2as'$, the speed v just before hitting the ground is given by $v^2 = (-5)^2 + 2(9.8)(3)$, so that $v = \sqrt{\frac{419}{5}}$

[When the ball passes through its original position on its downward path, its speed will also be $5ms^{-1}$ (by symmetry), so that alternatively $v^2 = (5)^2 + 2(9.8)(3)$.]

The speed after bouncing will be $e\sqrt{\frac{419}{5}}$

And, with upwards now being the positive direction,

$0^2 = e^2\left(\frac{419}{5}\right) + 2(-9.8)(3)$,

so that $e^2 = \frac{294}{419}$ and $e = 0.838$ (3sf)

Method 2

Let KE_0 & PE_0 be the initial kinetic & potential energies. Let KE_1 & KE_2 be the kinetic energies just before and after the bounce. And let PE_3 be the potential energy when the ball has regained its original height.

Then $KE_1 = KE_0 + PE_0$

Suppose that $KE_1 = \frac{1}{2}mv^2$. Then, $KE_2 = \frac{1}{2}m(ev)^2$,

so that $KE_2 = e^2KE_1$

Also $PE_3 = KE_2$, and as $PE_3 = PE_0$,

$PE_0 = e^2KE_1 = e^2(KE_0 + PE_0)$

$\Rightarrow mg(3) = e^2(\frac{1}{2}m(5)^2 + mg(3))$

$\Rightarrow e^2 = \frac{9.8(3)}{\left(\frac{25}{2}+9.8(3)\right)} = \frac{294}{419}$ and $e = 0.838$ (3sf)