Impulse & Momentum – Exercises (with Sol'ns)

(10 pages; 8/2/26)

(1)(i) A child of mass $40kg$ is standing on a stationary skateboard of mass $5kg$, and jumps off, so that his speed afterwards is $2ms^{-1}$ relative to the skateboard. What is the speed of the skateboard afterwards?

(ii) What impulse is given to the skateboard by the child?

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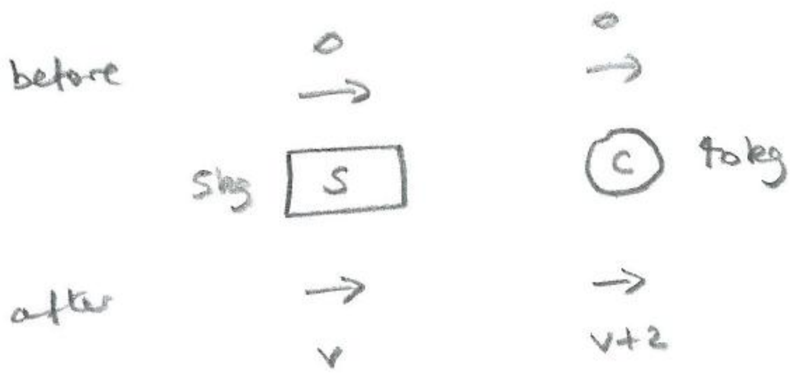
Solution

(i)



Conservation of momentum:

$0 = 5v + 40(v + 2)$

$\Rightarrow 45v = -80$

$\Rightarrow v = -\frac{80}{45} = -1.78\, ms^{-1}$ (3sf)

so that the skateboard has a speed of $1.78\, ms^{-1}$ (3sf)

(ii) Impulse given to the skateboard by the child, taking left to right as the positive direction: $5v - 5(0) = -\frac{80}{9}$

So impulse [to the left] is $\frac{80}{9} = 8.89\, Ns$ (3sf)

(2) A cat of mass $4kg$ is sitting on a stationary sledge of mass $8kg$. It then starts to walk along the sledge at a speed of $1ms^{-1}$, relative to the sledge. What happens to the sledge?

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Solution

The initial momentum of the system is zero.

Afterwards, let the speed of the sledge be v, in the direction that the cat walks.

Then the final momentum of the system is $8v + 4(v + 1)$

By conservation of momentum, $8v + 4(v + 1) = 0$,

so that $12v = -4$, and $v = -\frac{1}{3}$

ie the sledge moves at a speed of $\frac{1}{3}\ ms^{-1}$ in the opposite direction to that in which the cat initially walked.

(3) A ball is projected vertically upwards at a speed of $5ms^{-1}$ when it is $3m$ above the ground. Given that it just returns to its original height after bouncing on the ground (and assuming that there is no air resistance), find the coefficient of restitution between the ball and the ground. (Assume that $g = 9.8$)

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Solution

Method 1

Taking downwards to be the positive direction, and applying

$'v^2 = u^2 + 2as'$, the speed v just before hitting the ground is

given by $v^2 = (-5)^2 + 2(9.8)(3)$, so that $v = \sqrt{\frac{419}{5}}$

[When the ball passes through its original position on its downward path, its speed will also be $5ms^{-1}$ (by symmetry), so that alternatively $v^2 = (5)^2 + 2(9.8)(3)$.]

The speed after bouncing will be $e\sqrt{\frac{419}{5}}$

And, with upwards now being the positive direction,

$0^2 = e^2\left(\frac{419}{5}\right) + 2(-9.8)(3)$,

so that $e^2 = \frac{294}{419}$ and $e = 0.838$ (3sf)

Method 2

Let KE_0 & PE_0 be the initial kinetic & potential energies. Let KE_1 & KE_2 be the kinetic energies just before and after the bounce. And let PE_3 be the potential energy when the ball has regained its original height.

Then $KE_1 = KE_0 + PE_0$

Suppose that $KE_1 = \frac{1}{2}mv^2$. Then, $KE_2 = \frac{1}{2}m(ev)^2$,

so that $KE_2 = e^2 KE_1$

Also $PE_3 = KE_2$, and as $PE_3 = PE_0$,

$PE_0 = e^2 KE_1 = e^2(KE_0 + PE_0)$

$\Rightarrow mg(3) = e^2(\frac{1}{2}m(5)^2 + mg(3))$

$\Rightarrow e^2 = \frac{9.8(3)}{\left(\frac{25}{2}+9.8(3)\right)} = \frac{294}{419}$ and $e = 0.838$ (3sf)

(4) A spaceship has a geostationary orbit about the earth (ie it stays above the same point on the earth's surface). An astronaut walks from one end of the spaceship to the other. Describe what happens, relative to the earth's surface.

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Solution

Let the spaceship, excluding the astronaut, have mass M, and let the astronaut have mass m. Suppose that the astronaut is walking with velocity w relative to the spaceship, and that the spaceship (including the astronaut) travels at velocity v relative to the earth's surface, once the astronaut has started walking.

By conservation of momentum,

$Mv + m(v + w) = 0 \quad \Rightarrow v(M + m) = -mw$

and so $v = -\frac{mw}{(M+m)}$

ie the spaceship moves in the opposite direction to the motion of the astronaut relative to the spaceship.

Consider the motion of the centre of mass of the spaceship and astronaut.

Its velocity relative the the earth's surface is the weighted average of the velocities of the spaceship (excluding the astronaut) and the astronaut:

$\left(\frac{M}{M+m}\right) v + \left(\frac{m}{M+m}\right)(v + w)$

$= \left(\frac{1}{M+m}\right)(Mv + mv + mw)$

$= v + \frac{mw}{M+m} = 0$

As there is no external force on the spaceship and astronaut, we would expect there to be no net motion of the centre of mass of the spaceship and astronaut.