Important Ideas - Transformations (3 pages; 22/10/20)

(1) Translation of $\binom{a}{b}$: $y = f(x) \rightarrow y - b = f(x - a)$

(2) Stretch of scale factor k in the x direction (eg if $k = 2$, graph of $y = x^2$ is stretched outwards, so that the x-coordinates are doubled): $y = f(x) \rightarrow y = f(\frac{x}{k})$

Stretch of scale factor k in the y direction: $y = f(x) \rightarrow \frac{y}{k} = f(x)$

(3) Reflection in the y-axis: $f(x) \rightarrow f(-x)$

Reflection in the x-axis: $y = f(x) \rightarrow -y = f(x)$

(4) Note that, at each stage of a composite transformation, we must be replacing x by either $x + a$ (where a can be negative) or kx (and similarly for y).

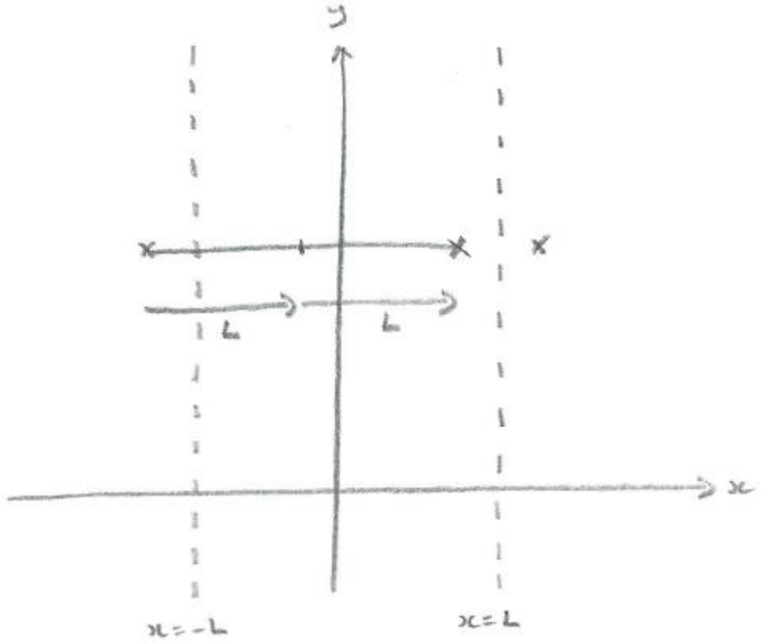
(5) Reflection in the line $x = L$: $y = f(x) \rightarrow y = f(2L - x)$

Reflection in the line $y = M$: $y = f(x) \rightarrow 2M - y = f(x)$

Justification

A reflection in the line $x = L$ is equivalent to a reflection in the line $x = 0$, followed by a translation of $\binom{2L}{0}$ (see the diagram below).

Thus $y = f(x) \rightarrow y = f(-x) \rightarrow y = f(-[x - 2L]) = f(2L - x)$



Similarly for a reflection in the line $y = M$, so that

$y = f(x) \rightarrow -y = f(x) \rightarrow -(y - 2M) = f(x)$ or $2M - y = f(x)$

(6) Example: To obtain $y = \sin(2x + 60)$ from $y = sinx$,

either (a) stretch by scale factor $\frac{1}{2}$ in the x direction, to give

$y = \sin(2x)$, and then translate by $\binom{-30}{0}$, to give

$y = \sin(2[x + 30]) = \sin(2x + 60)$

or (b) translate by $\binom{-60}{0}$, to give $y = \sin(x + 60)$, and then

stretch by scale factor $\frac{1}{2}$ in the x direction, to give

$y = \sin(2x + 60)$ [It is perhaps more awkward to produce a sketch by method (b).]

[Note that, at each stage, we are either replacing x by kx, or by

$x \pm a$]

(7) Transformations involving moduli signs

(i) $y = f(|x|)$

$f(|x|) = f(x)$ when $x \geq 0$

$f(|x|) = f(-x)$ when $x < 0$ (ie the left-hand half of $y = f(x)$ is replaced by the reflection of the right-hand half in the y-axis)

(ii) $|y| = f(x)$

As $|y| \geq 0$, the graph is undefined where $f(x) < 0$.

Where $f(x) \geq 0$, the graph of $|y| = f(x)$ is that of $y = f(x)$, together with its reflection in the x-axis.

(8) A rotation of 180° is equivalent to a reflection in the line

$x = 0$, followed by a reflection in the line $y = 0$, so that $y = f(x)$

$\rightarrow y = -f(-x)$