Important Ideas - Turning Points & Points of Inflexion

(2 pages; 4/1/21)

Turning Points

(1) $\frac{d^2y}{dx^2} \neq 0$ is a sufficient (but not necessary) condition for a turning point (eg $\frac{d^2y}{dx^2} = 0$ at $x = 0$ for $y = x^4$)

(2) A necessary and sufficient condition for a turning point is that the 1st non-zero derivative of the function should be of even order (and ≥ 2) (eg $y = x^4$, where $\frac{dy}{dx} = \frac{d^2y}{dx^2} = \frac{d^3y}{dx^3} = 0$, but $\frac{d^4y}{dx^4} \neq 0$)

(3) To find the turning points of $y = \frac{x^2-2x+2}{x^2-3x-4}$, consider the quadratic $\frac{x^2-2x+2}{x^2-3x-4} = k$, with $b^2 - 4ac = 0$ (to give a quadratic in k).

Points of Inflexion

(1) A point of inflexion occurs at the turning point of the gradient. A turning point occurs when the gradient changes sign (either from positive to negative, in the case of a maximum, or from negative to positive, in the case of a minimum). So a point of inflexion occurs when the gradient of the gradient changes sign; ie when $\frac{d^2y}{dx^2}$ changes sign.

(2) A point of inflexion need not be a stationary point (ie where $\frac{dy}{dx} = 0$); eg $y = sinx$ at $x = 0$

(3) $\frac{d^2y}{dx^2} = 0$ is a necessary (but not sufficient) condition for a point of inflexion (e.g. $\frac{d^2y}{dx^2} = 0$ at $x = 0$ for $y = x^4$, but there is no point of inflexion)

(4) $\frac{d^2y}{dx^2} = 0$ and $\frac{d^3y}{dx^3} \neq 0$ is a sufficient (but not necessary) condition for a point of inflexion (eg $y = x^5$, which has a point of inflexion at at $x = 0$, but $\frac{d^3y}{dx^3} = 0$)

(5) A necessary and sufficient condition for a point of inflexion is that the first non-zero derivative of the function, excluding $\frac{dy}{dx}$, should be of odd order (eg $y = x^5$ where $\frac{d^3y}{dx^3} = 0$ and $\frac{d^5y}{dx^5} \neq 0$)

(6) At a point of inflexion, the curve changes from being concave ($\frac{d^2y}{dx^2} < 0; eg\ y = lnx$) to convex ($\frac{d^2y}{dx^2} > 0; eg\ y = e^x\ [conve^x]$), or vice-versa.

(7) A polynomial function of the form

$y = (x - a)^{2m}(x - b)^{2n+1}$ ... has a turning point at $(a, 0)$ and a point of inflexion at $(b, 0)$, when $m > 0$ & $n > 0$