

Important Ideas - Quadratic Functions (2 pages; 22/10/20)

(1) Quadratic curves

Example: $y = x^2 - 2x - 3$

$$x^2 - 2x - 3 = (x + 1)(x - 3)$$

Also $x^2 - 2x - 3 = (x - 1)^2 - 4$

The minimum point of $(1, -4)$ lies on the line of symmetry of the curve, which is equidistant from the two roots of $x^2 - 2x - 3 = 0$: -1 & 3 .

Also, from the quadratic formula (which is itself derived by completing the square on $ax^2 + bx + c$):

$$x = \frac{2 \pm \sqrt{4 + 12}}{2} = 1 \pm 2$$

Thus the roots of $x^2 - 2x - 3 = 0$ lie the same distance either side of the line of symmetry of the curve.

(2) Factorisation of quadratics

Example : $f(x) = 6x^2 + x - 12$

We need to find A and B such that $A + B = 1$ (the coefficient of x) and $AB = -72$ (the product of the coefficient of x^2 and the constant term)

$A = 9$ and $B = -8$ satisfy this

Then $f(x) = 6x^2 + 9x - 8x - 12$

$$= 3x(2x + 3) - 4(2x + 3)$$

$$= (3x - 4)(2x + 3)$$

$$\begin{aligned}\text{Alternatively, } f(x) &= 6x^2 - 8x + 9x - 12 \\ &= 2x(3x - 4) + 3(3x - 4) \\ &= (2x + 3)(3x - 4)\end{aligned}$$