

Important Ideas - Logarithms (2 pages; 22/10/20)

$$(1) \log_a b = c \Leftrightarrow a^c = b$$

$$(2) \text{ eg } 3 + 2\log_2 5 = 3\log_2 2 + \log_2(5^2)$$

$$= \log_2(2^3) + \log_2(5^2) = \log_2(8 \times 25) = \log_2(200)$$

$$(3) \log_a b \log_b c = \log_a c \quad \text{or} \quad \log_b c = \frac{\log_a c}{\log_a b}$$

Proof: Let $b = a^x$ & $c = b^y$

$$\text{Then } c = (a^x)^y = a^{xy}$$

$$\text{and } \log_a c = xy = \log_a b \log_b c$$

$$\text{Special case: } \log_b c = \frac{1}{\log_c b}$$

MAT/STEP:

(4) As $\log_8 8 = 1$ and $\log_8 64 = 2$, and as $y = \log_8 x$ is a concave function ($\frac{dy}{dx}$ is decreasing; ie $\frac{d^2y}{dx^2} < 0$), linear interpolation

$$\Rightarrow \log_8 \left[\frac{1}{2}(8 + 64) \right] > \frac{1}{2}(1 + 2)$$

$$\text{ie } \log_8 36 > \frac{3}{2}$$

(5) To find an upper bound for eg $\log_2 3$:

Suppose that $\log_2 3 < \frac{m}{n}$

Then $3 < 2^{\left(\frac{m}{n}\right)}$ and $3^n < 2^m$

As $243 = 3^5 < 2^8 = 256$, $\log_2 3 < \frac{8}{5}$

[and $\frac{8}{5}$ is a reasonably low upper bound, as 243 & 256 are reasonably close]

(6) eg $\log_2 12 = \log_2(3 \times 4) = \log_2 3 + \log_2 4 < \frac{8}{5} + 2 = \frac{18}{5}$,

from (5)

(7) eg $\log_{36} 8 = \frac{1}{\log_8 36} < \frac{2}{3}$, from (4)

(8) Example: Show that $\log_5 10 < \frac{3}{2}$

$\log_5 10 < \frac{3}{2} \Leftrightarrow 10 < 5^{\left(\frac{3}{2}\right)}$ (as the log function is increasing)

$\Leftrightarrow 10^2 < 5^3 \Leftrightarrow 100 < 125$