

Important Ideas - Integers (1 page; 22/10/20)

(1) Tests for divisibility

(1.1) If the sum of the digits of a number is a multiple of 3, then the number itself is a multiple of 3; and similarly for 9.

$$(1.2) 11 \times 325847 = 3584317$$

and $3 - 5 + 8 - 4 + 3 - 1 + 7 = 11$, which is a multiple of 11

This is true in all cases: If $a - b + c - d + \dots - z$ is a multiple of 11, then $abcd \dots z$ is a multiple of 11.

[and also for $a - b + c - d + \dots + y$]

(2) Factorisations [MAT/STEP]

Let $f(n)$ be the number of factors of n (including 1).

If $n = pq$, where p & q have no common factors (other than 1), then $f(n) = f(p)f(q)$.

[eg $100 = 2^2 \times 5^2$; factors are obtained from $\{1, 2, 4\}$ with $\{1, 5, 25\}$, giving a total of $3 \times 3 = 9$ factors: 1, 5, 25, 2, 10, 50, 4, 20, 100]

(3) Integer solutions [MAT/STEP]

$$\text{eg } xy - 8x + 6y = 90$$

$$\text{can be rearranged to } (x + 6)(y - 8) = 42$$