Important Ideas - Inequalities (2 pages; 22/10/20)

(1) Beware of multiplying inequalities by a quantity that is (or could be) negative (eg log(0.5)).

(2) Inequalities can often be solved by considering the critical value(s) where equality holds.

(3) If a and b are ≥ 0, then $a > b \Leftrightarrow a^2 > b^2$ (as $y = x^2$ is an increasing function for $x \geq 0$).

(4) If an expression can be arranged into the form $(a - b)^2$, then this will be non-negative.

(5) Methods for solving $\frac{x+1}{x-2} < 2x$

Method 1: Multiply both sides by $(x-2)^2$ (as this is positive, assuming that $x \neq 2$). The resulting cubic will have a factor of

$x - 2$. Consider the regions of the graph.

Method 2: Treat the cases $x - 2 < 0$ and $x - 2 > 0$ separately

Method 3: Rearrange as $\frac{x+1}{x-2} - 2x \ < 0$, and write the LHS as a single fraction. Consider the critical points where either the numerator or the denominator is zero.

Method 4: Sketch $y = \frac{x+1}{x-2}$ and $y = 2x$, and consider the points of intersection.

(6) $|x-2|>5$

Method 1

x is more than 5 away from 2, and so has to be either <-3 or >7

Method 2

$|x-2|>5 \Leftrightarrow (x-2)^2>25$ etc

Method 3

Case 1: $x-2\geq 0$; Case 2: $x-2<0$

Method 4

Draw graphs of $y=|x-2|$ and $y=5$

(7) $2<|x+3|<7$

Method 1 Distance of x from -3 is between 2 and 7

So $-10<x<-5\ or -1<x<4$

Method 2 $2<|x+3|<7 \Leftrightarrow 4<(x+3)^2<49$ etc

Method 3 Case 1: $x+3\geq 0$; Case 2: $x+3<0$

(8) $|x-2|>|x-5|$

Method 1 x has to be further from 2 than from 5.

It is equidistant when $x=\frac{7}{2}$, and so $x>\frac{7}{2}$.

Method 2 $|x-2|>|x-5| \Leftrightarrow (x-2)^2>(x-5)^2$ etc

Method 3 Case 1: $x<2$; Case 2: $2\leq x<5$; Case 3: $x\geq 5$

Method 4 Draw the graphs of $y=|x-2|$ and $y=|x-5|$