Important Ideas - Functions (2 pages; 22/10/20)

(1) Convex & concave functions

An example of a convex function is $y = e^x$ (think of conve^x!).

For a convex function, the line connecting any two points on the curve lies entirely above the curve. Also, a function is convex when $\frac{d^2y}{dx^2} > 0$.

An example of a concave function is $y = lnx$. For a concave function, the line connecting any two points on the curve lies entirely below the curve. Also, a function is concave when

$\frac{d^2y}{dx^2} < 0$.

A point of inflexion occurs when $\frac{d^2y}{dx^2}$ changes sign. This is when a function changes from convex to concave (or the reverse).

(2) Curve Sketching [MAT/STEP]

(2.1) Symmetry about $x = a$: $f(a - \lambda) = f(a + \lambda)$ for all λ

[Special case: symmetry about the y-axis: $f(-x) = f(x)$]

Alternatively, $f(2a - x) = f(x)$ for all x [setting $x = a + \lambda$]

Example: $\sin(\pi - \theta) = sin\theta$, and the sine curve has symmetry about $\theta = \frac{\pi}{2}$

(2.2) If you are asked to sketch a curve defined for $x \in [a, b]$, consider whether it might have symmetry about the mid-point $\frac{a+b}{2}$.

STEP:

(2.3) For $y = |f(x)|$, when $f(x) = 0$, there will be a cusp.

Note when sketching the curve that $f'(x_0 + \delta) = -f'(x_0 - \delta)$.

(3) Greatest or least value of a function [STEP]

(3.1) Beware of establishing the greatest or least value of a function from stationary points: these only indicate local maxima and minima.

Also, a greatest or least value may occur at a boundary of the domain.

(3.2) Possibilities for demonstrating that $f(x) \geq 0$

(i) $f(x) = [g(x)]^2 + [h(x)]^2$ (for all x)

(ii) For $x \geq a$: establish that $f(a) \geq 0$ and that $f'(x) \geq 0$

for $x \geq a$.

(iii) $f(x) = xsinhx[g(x)]^2$ (as x & $sinhx$ will always have the same sign - unless they are both zero) (for all x)