Important Ideas - Complex Numbers (2 pages; 22/10/20)

(1) If $z = a + bi$, then the imaginary part of z is b (not bi).

For this reason, the positive imaginary axis of the Argand diagram should be labelled 1, 2, ... (although $i, 2i, \ldots$ is seen in some textbooks).

An imaginary number can be defined as any complex number on the imaginary axis; ie a multiple of i (including zero).

All numbers in the Argand diagram (including real numbers and imaginary numbers) are 'complex numbers'. The number $2 + 3i$ can be referred to as a non-real complex number, if necessary.

(2) When r is a non-negative real number, $\sqrt{r}$ is defined to be the positive square root (so that the solutions of $x^2 = r$ are $x = \pm\sqrt{r}$).

Because complex numbers are represented by points in the Argand diagram (in contrast to real numbers, which are represented by points on a number line), multiplication by -1 has a more complicated interpretation; namely as a rotation of 180°.

The square root of the complex number $z = re^{i\theta}$

($r \geq 0$ & $-\pi < \theta \leq \pi$) is defined as $\sqrt{z} = \sqrt{r}e^{i\theta/2}$, and the solutions of $u^2 = z$ are $u = \pm\sqrt{r}e^{i\theta/2}$.

However, the complex square root function is not continuous, as when $z = e^{i\pi}, \sqrt{z} = e^{i\pi/2} = i$, whilst for the neighbouring point in the Argand diagram, $z = e^{-i(\pi-\delta)}, \sqrt{z} = e^{-i(\pi-\delta)/2}$, which is close to $e^{-i\pi/2} = -i$. It can be shown that, for this reason, it is not generally true that $\sqrt{uv} \neq \sqrt{u}\sqrt{v}$. For example, $\sqrt{-1}\sqrt{-1} = i^2 = -1$, but $\sqrt{(-1)(-1)} = \sqrt{1} = 1$.

(3) Complex numbers are extremely versatile. They can be treated as vectors, or as points (as well as rotations) in the Argand diagram (often leading to a problem in geometry), or simply as numbers (manipulated in a similar way to real numbers). [See STEP: "Complex Number Methods".]