

Important Ideas - Algebra (2 pages; 22/10/20)

(1) Equating coefficients

Example: To divide $f(x) = x^3 + x^2 - 11x + 10$ by $x - 2$

First of all, $f(2) = 8 + 4 - 22 + 10 = 0$, so that there is no remainder.

$$\text{Then } x^3 + x^2 - 11x + 10 = (x - 2)(x^2 + ax - 5)$$

Equating coefficients of x^2 : $1 = a - 2$, so that $a = 3$

(Check: Equating coefficients of x : $-11 = -5 - 2a$, so that $a = 3$)

This method is usually quicker than long division.

(2) Expansions [MAT/STEP]

$$(i) (a + b + c)^2 = (a^2 + b^2 + c^2) + 2(ab + ac + bc)$$

$$(ii) (a + b + c)^3 = (a^3 + b^3 + c^3)$$

$$+ 3(a^2b + a^2c + b^2a + b^2c + c^2a + c^2b)$$

$$+ 6abc$$

STEP:

$$(iii) (a + b + c)^4 = (a^4 + b^4 + c^4)$$

$$+ 4(a^3b + a^3c + b^3a + b^3c + c^3a + c^3b)$$

$$+ 6(a^2b^2 + a^2c^2 + b^2c^2) + 12(a^2bc + b^2ac + c^2ab)$$

$$(iv) (a + b + c)^n = \sum_{\substack{i,j,k \\ (i+j+k=n)}} \binom{n}{i,j,k} a^i b^j c^k,$$

$$\text{where } \binom{n}{i,j,k} = \frac{n!}{i!j!k!}$$