Hyperbolas - Exercises (2 pages; 17/2/20)

Key to difficulty:

* easier

** moderate

*** harder

(1**) Show that the equation of the tangent to the hyperbola

$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ at the point $(acosht, bsinht)$ is

$yasinht = xbcosht - ab$

(2***) Given that the tangent in (1) meets the asymptotes of the hyperbola at the points P & Q, show that the mid-point of P & Q is $(acosht, bsinht)$.

(3***) In the case where $b = a$, find the area of the triangle OPQ (where P & Q are as in (2), and O is the Origin).

(4***) The chord PQ, where P and Q are points on the rectangular hyperbola $xy = c^2$, has gradient 1. Show that the locus of the point of intersection of the tangents from P and Q is the line

$y = -x$. [Edx FP3 textbook, Ex. 2G, Q9]

(5***) Use matrices to show that the rectangular hyperbola

$x^2 - y^2 = a^2$ can be obtained by rotating the rectangular hyperbola $xy = c^2$, expressing a^2 in terms of c.

(6**) Show that the equation of the normal to the hyperbola

$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ at the point $(acosht, bsinht)$ is

$xasinht + ybcosht = (a^2 + b^2)sinhtcosht$

(7***) [Edx (Pearson), FP1, ME2, Q7 (p58)]

l_1& l_2 are distinct tangents to the rectangular hyperbola $xy = 9$ with gradient $-\frac{1}{4}$; find the equations of l_1& l_2

(8***) [Edx (Pearson), FP1, ME2, Q14 (p59)]

Suppose that P is a general point on a rectangular hyperbola and that the tangent at P crosses the x and y axes at A and B respectively. Show that:

(i) AP=PB

(ii) the triangle OAB has a constant area, as P varies