# Hooke's Law – Q4 [6 marks] (4/6/21)

## Exam Boards

OCR : Mechanics (Year 2)

MEI:  Mechanics b

AQA: Mechanics (Year 1)

Edx: Mechanics 1 (Year 2)

Two elastic strings AB and BC are joined together at B, to form one long string. String AB has natural length $4m$ and modulus of elasticity $20N$; string BC has natural length $2m$ and modulus of elasticity $30N$. The ends A and C of the long string are attached to two fixed points which are $10m$ apart. Find the tension in the combined string. [6 marks]

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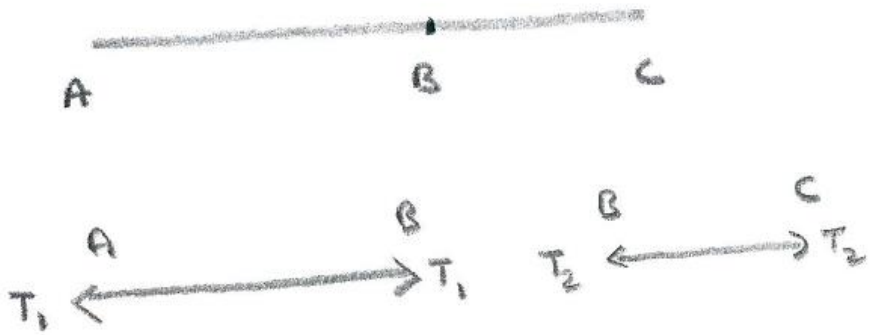
## Solution




[Considering the force diagram for AB: by N2L, the reaction at A will equal the force applied by string BC at B. Call this $T_1$ - this is the tension that AB is under. Similarly, string BC will be under tension $T_2$. By N3L, the forces that the two strings apply to each other will be equal and opposite, so that $T_1 = T_2 = T$, say. The tension in the combined string (determined by the reactions at A and C) will therefore be $T$ also.]

For string AB, Hooke's law $\Rightarrow T_1 = \frac{20e_1}{4}$, where $e_1$ is the extension of string AB. [1 mark]

Similarly, for string BC, $T_2 = \frac{30e_2}{2}$ [1 mark]

Also $(4 + e_1) + (2 + e_2) = 10$, so that $e_1 + e_2 = 4$ [1 mark]

Then $T_1 = T_2 \Rightarrow \frac{20e_1}{4} = \frac{30e_2}{2}$, so that $5e_1 = 15(4 - e_1)$, [1 mark]

and $e_1 = 3(4 - e_1)$, so that $4e_1 = 12$ and hence $e_1 = 3$, and $e_2 = 1$. [1 mark]

Therefore $T = 5e_1 = 15N$ [1 mark]

**Note:** A result [which would probably have to be proved in an exam, though possibly not for STEP] that can be applied here is that, with strings of stiffness $k_1$ and $k_2$ in series, the combined string has stiffness $k$ given by $\frac{1}{k} = \frac{1}{k_1} + \frac{1}{k_2}$

In this case, $\frac{1}{k} = \frac{4}{20} + \frac{2}{30} = \frac{16}{60}$, so that $k = \frac{15}{4}$,

and $T = k(e_1 + e_2) = \frac{15}{4}(4) = 15N$