Hooke's Law – Q2 [18 marks] (4/6/21)

Exam Boards

OCR : Mechanics (Year 2)

MEI: Mechanics b

AQA: Mechanics (Year 1)

Edx: Mechanics 1 (Year 2)

A particle of mass 200g hangs at a point Q, suspended from a fixed point P, by means of a spring of original length 20cm and modulus of elasticity 5N. It is pulled down to a point R, which is 35cm below P. The particle is then released.

Ignoring any resistances to motion, find:

(i) the work done in pulling the particle down to R [7 marks]

(ii) the maximum speed of the particle after it is released, and the point at which this occurs [4 marks]

(iii) the distance of the particle below P when it reaches its maximum height, at position S, and show that the distance QS equals the distance QR [7 marks]

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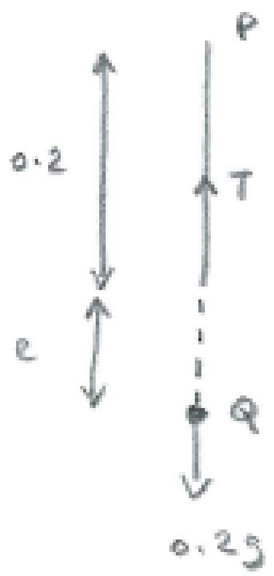
Solution



[Note: The g in the diagram (gravity) is not to be confused with g for grams.]

(i) If e is the extension of the spring at Q (in metres), Hooke's Law $\Rightarrow \frac{\lambda e}{l} = T = mg \Rightarrow \frac{5e}{0.2} = (0.2)(9.8) \Rightarrow e = 0.0784$

[2 marks]

Taking the zero of gravitational potential energy (GPE) to be R,

the total energy of the particle at Q is:

GPE + EPE + KE (where EPE is elastic potential energy & KE is kinetic energy)

$= (0.2)(9.8)(0.35 - 0.2 - 0.0784) + \frac{1}{2}\left(\frac{5}{0.2}\right)(0.0784)^2 + 0$

$= 0.140336 + 0.076832 + 0 = 0.217168$ [3 marks]

The total energy of the particle at R is:

$0 + \frac{1}{2}\left(\frac{5}{0.2}\right)(0.15)^2 + 0 = 0.28125$ [1 mark]

Thus the work done $= 0.28125 - 0.217168 = 0.064082 =$ 0.0641 J (3sf) [1 mark]

(ii) The maximum speed will occur when the particle is not accelerating; ie at Q, where the net force on the particle is zero [as $T = mg$ at the equilibrium position]. [1 mark]

The KE of the particle at Q will equal the work done to pull it down to R, as this is the energy gained by the particle since it was last at Q.   [1 mark]

Hence $\frac{1}{2}(0.2)v^2 = 0.064082$ (where v is the maximum speed)

and $v = 0.80051 = 0.801\ ms^{-1}$ (3sf)   [2 marks]

(iii) Let d be the distance below P when the particle is at S.

The total energy of the particle at S is:

$$(0.2)(9.8)(0.35-d)+\frac{1}{2}\left(\frac{5}{0.2}\right)(d-0.2)^2+0 \quad [2 \text{ marks}]$$

and this equals the energy at R of 0.28125 ,so that

$$12.5d^2-6.96d+0.90475=0 \; [1 \text{ mark}]$$

and $d=\frac{6.96\pm\sqrt{3.2041}}{25}=0.35 \; or \; 0.2068$

Thus 0.35 corresponds to R and S is the point 20.68cm below P.

[2 marks]

This is $20+7.84-20.68=7.16$ cm above the equilibrium position Q, whilst R is $35-(20+7.84)=7.16$ cm below Q.

[2 marks]

[The particle oscillates between R and S.]