Hooke's Law – Q1 [10 marks] (4/6/21)

Exam Boards

OCR : Mechanics (Year 2)

MEI: Mechanics b

AQA: Mechanics (Year 1)

Edx: Mechanics 1 (Year 2)

A particle of mass 200g is attached at the mid-point of an elastic string of natural length 0.5m and modulus of elasticity λ, which hangs vertically between two points, 1m apart.

(i) How far will the particle be below the top point if $\lambda = 1$?

[6 marks]

(ii) Determine the minimum value of λ such that there is no slack in the string. [4 marks]

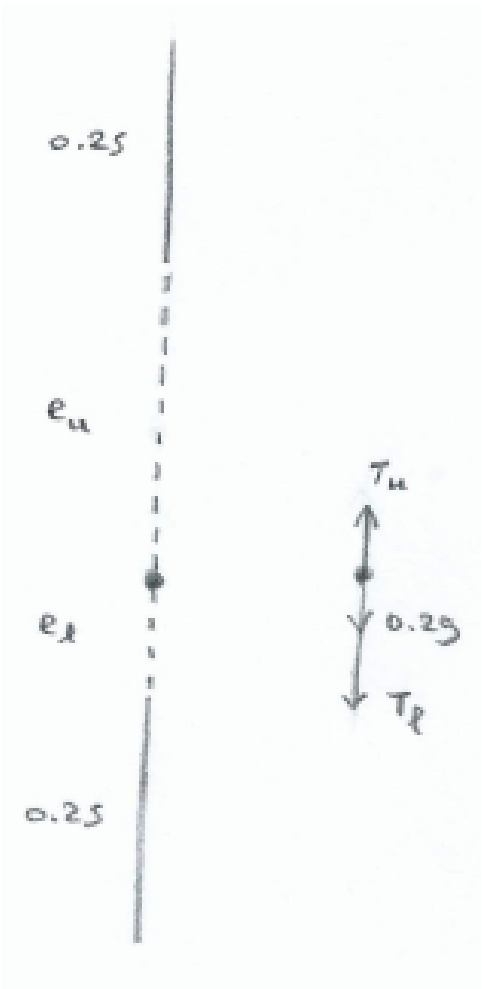
A particle of mass 200g is attached at the mid-point of an elastic string of natural length 0.5m and modulus of elasticity λ, which hangs vertically between two points, 1m apart.

(i) How far will the particle be below the top point if $\lambda = 1$?

[6 marks]

(ii) Determine the minimum value of λ such that there is no slack in the string. [4 marks]

Solution



(i) Let the extensions of the upper and lower parts of the string be e_u and e_l, respectively, and the tensions in the two parts T_u and T_l.

Then, referring to the diagram,

$T_u = \frac{\lambda e_u}{0.25}$, $T_l = \frac{\lambda e_l}{0.25}$ (assuming the string is not slack) [1 mark]

Equilibrium $\Rightarrow T_u = T_l + 0.2g$ [1 mark]

Also $e_u + e_l = 0.5$ (1) [1 mark]

Hence $\lambda e_u = \lambda e_l + 0.05g$ [1 mark]

and so $\lambda e_u = \lambda(0.5 - e_u) + 0.05g$,

giving $2\lambda e_u = 0.5\lambda + 0.05g$ [1 mark]

and hence $e_u = \frac{0.5\lambda + 0.05g}{2\lambda}$ (2)

Thus when $\lambda = 1$, $e_u = 0.495$

and the distance below the top point is $0.25 + 0.495 = 0.745m$

[1 mark]

(ii) The string is slack if $e_l < 0$ [1 mark]

From (1) & (2), $e_l = 0.5 - 0.25 - \frac{0.025g}{\lambda} = 0.25 - \frac{0.025g}{\lambda}$ [1 mark]

Thus we require $0.25 - \frac{0.025g}{\lambda} \geq 0$,

so that $0.25 \geq \frac{0.025g}{\lambda}$ and $\lambda \geq 0.1g = 0.98$ [2 marks]