

Graphs Glossary (15/1/2014)

Graphs have either **Nodes & Arcs** or (less commonly) **Vertices & Edges**. (These become **Events & Activities** in Critical Path Analysis.)

Nodes are of odd or even **order** (or **degree** or **valency**) - depending on the number of arcs meeting at the node.

A **connected** graph is one for which a path can be found between any two nodes.

A **simple** graph is one with no multiple arcs [joining different nodes] or loops [arcs joining a node to itself]

A **walk** is a sequence of arcs (where the end of one arc is the start of the next - except for the last arc); arcs and nodes can be repeated.

A **trail** (aka **route**): eg DBCADAB (a walk with no arcs repeated; though nodes can be repeated)

[Note: It isn't possible to have a walk where nodes aren't repeated, but arcs **are** repeated.]

Closed trail: initial and final nodes are the same, but nodes can be repeated; eg BCADAB

Path: A walk with no arcs **or** nodes repeated; eg DBCA

Cycle: Closed path; eg BCADB

An **Eulerian graph** has a closed trail covering each arc exactly once. [A graph is Eulerian if (and only if) every node has even order.]

A graph is **semi-Eulerian** if it has a trail which is not closed that contains every arc exactly once (ie not returning to the starting point). [A graph is semi-Eulerian if (and only if) exactly two nodes have odd order.]

[There must be an even number of nodes of odd order.]

A **traversable** graph is one that is either Eulerian or semi-Eulerian (the graph can be traced without lifting the pen from the page).

Eulerian trail/path/cycle: where arcs are traversed exactly once

Hamiltonian path/cycle A path/cycle that visits each node exactly once. [A Hamiltonian cycle is sometimes referred to as a **tour**]

Summary

| | |
|--------------------------|--|
| walk | both arcs and nodes can be repeated |
| trail (aka route) | arcs can't be repeated, but nodes can be |
| path | neither arcs nor nodes can be repeated |
| cycle | a path where the starting point is returned to |
| tour (Hamiltonian cycle) | a cycle which visits each node exactly once (some arcs may not be traversed) |
| Eulerian cycle | a cycle where arcs are traversed exactly once (and nodes are visited exactly once) |

A **planar** graph is one where the arcs don't cross (ie arcs only meet at nodes) - or which can be redrawn so that they don't.

A **tree** is a connected graph with no cycles.

A **subgraph** is a subset of the nodes and arcs of a particular graph.

A **spanning tree** is a subgraph which contains all the nodes of the graph, and which is a tree.

A **Minimum Connector** is a spanning tree of minimum possible total weight.

Complete graph: every node is joined exactly once to every other node; eg K_5 (5 nodes).

Digraph: One or more arcs can only be traversed in a particular direction.

Bipartite graph: There are two sets of nodes (eg one set on the left-hand side, and one on the right-hand side), and the arcs only connect nodes in one set with nodes in the other.

A **network** is a graph with **weights** attached to the arcs.