Graphs - Exercises (with Sol'ns) (7 pages; 25/2/26)

(1) Prove that the number of arcs in K_n is $\frac{1}{2}(n-1)n$?

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Solution

Method 1

There are n nodes of order $n-1$

So total of orders $= n(n-1)$,

and total number of arcs $= \frac{1}{2}n(n-1)$

Method 2

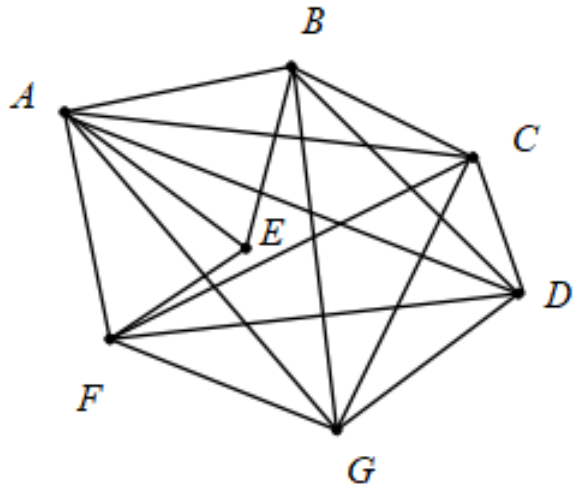
There are $n-1$ arcs joining the 1st node.

After the 1st node has been excluded, there are a further $n-2$ arcs joining the 2nd node; and so on.

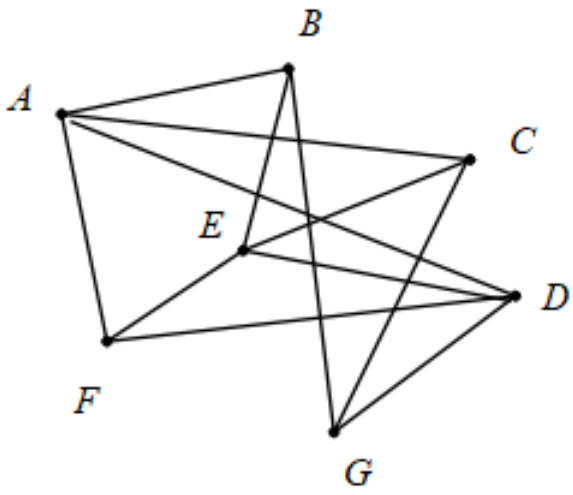
Giving a total of $(n-1)+(n-2)+\cdots+1 = \frac{1}{2}(n-1)n$

(2) Use Kuratowski's theorem to decide whether the following graphs are planar.

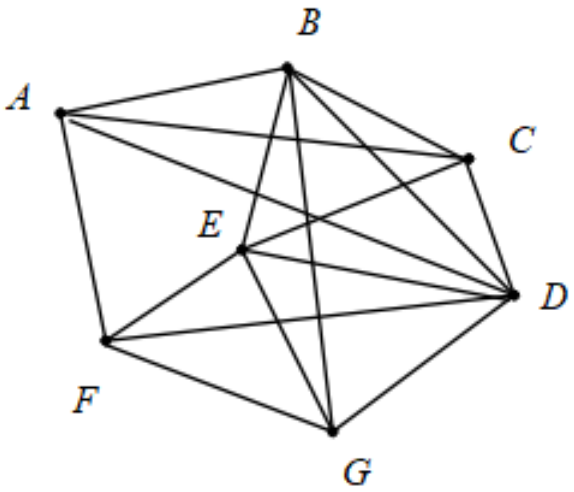
(i)



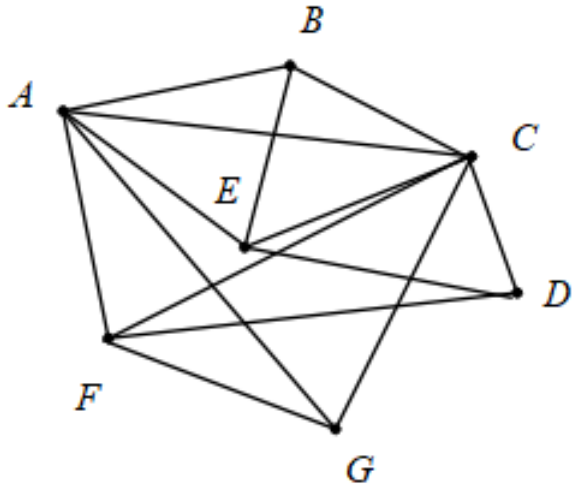
(ii)



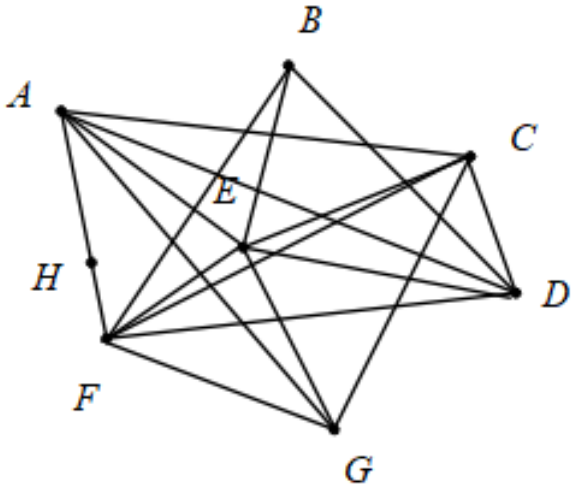
(iii)



(iv)



(v)



Solution

(i) A subgraph of this graph is ACDFG, which is K_5, and so the graph is non-planar.

(ii) A subgraph of this graph is AEGBCD, which is $K_{3,3}$, and so the graph is non-planar.

(iii) Neither K_5 nor $K_{3,3}$ (or a sub-division of these) is a subgraph of this graph, and so the graph is planar.

(iv) Neither K_5 nor $K_{3,3}$ (or a sub-division of these) is a subgraph of this graph, and so the graph is planar.

(v) A subgraph of this graph is a sub-division of ACDEF, which is K_5, and so the graph is non-planar.

(3) Use the Planarity algorithm to determine whether or not K_5 is planar.

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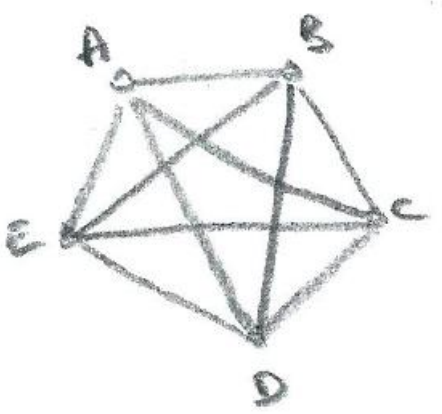
Solution



ABCDE is a Hamiltonian cycle

The interior arcs are AC, AD, BD, BE & CE

Give AC the label I [to indicate that it is to be an interior arc]

BD & BE are the arcs that cross AC, and they don't cross each other, and so can be labelled O

AD & CE each cross one (or more) of BD & BE, but cross each other, and so can't be labelled I

Hence the graph is non-planar