

Geometric Distribution Q1 [10 marks] (10/6/21)

Exam Boards

OCR : Statistics (Year 1)

MEI: Statistics a

AQA: -

Edx: S1 (Year 2)

Repeated independent trials of an experiment are carried out. On each trial the probability of success is $\frac{1}{10}$.

(i) Find the probability that the 1st success occurs after the 6th trial. [2 marks]

(ii) Find the probability that the 3rd success occurs on the 6th trial. [3 marks]

(iii) Find the smallest value of n such that the probability of at least one success in n trials is more than $\frac{9}{10}$. [5 marks]

Repeated independent trials of an experiment are carried out. On each trial the probability of success is $\frac{1}{10}$.

(i) Find the probability that the 1st success occurs after the 6th trial. [2 marks]

(ii) Find the probability that the 3rd success occurs on the 6th trial. [3 marks]

(iii) Find the smallest value of n such that the probability of at least one success in n trials is more than $\frac{9}{10}$. [5 marks]

Solution

(i) Probability = Probability that 1st 6 trials are failures [1 mark]

$$= \left(\frac{9}{10}\right)^6 = 0.53144 = 0.531 \text{ (3sf) [1 mark]}$$

(ii) Probability = Probability that there are 2 successes in the 1st 5 trials, and then a success on the 6th trial [1 mark]

$$= \binom{5}{2} \left(\frac{1}{10}\right)^2 \left(\frac{9}{10}\right)^3 \times \frac{1}{10} = \frac{9^3}{10^5} = 0.00729 \text{ [2 marks]}$$

(iii) Probability of at least one success in n trials

$$= 1 - \text{Prob}(\text{no successes in } n \text{ trials}) \text{ [1 mark]}$$

$$= 1 - \left(\frac{9}{10}\right)^n$$

$$\text{Then } 1 - \left(\frac{9}{10}\right)^n = \frac{9}{10} \text{ [1 mark]}$$

$$\Rightarrow \left(\frac{9}{10}\right)^n = \frac{1}{10}$$

$$\Rightarrow n \ln(0.9) = \ln(0.1) \text{ [1 mark]}$$

$$\Rightarrow n = 21.854 \text{ [1 mark]}$$

So n needs to be 22. [1 mark]