

Geometric Distribution - Exercises (with Sol'ns)

(5 pages; 14/2/26)

(1) Repeated independent trials of an experiment are carried out. On each trial the probability of success is $\frac{1}{10}$.

(i) Find the probability that the 1st success occurs after the 6th trial.

(ii) Find the probability that the 3rd success occurs on the 6th trial.

(iii) Find the smallest value of n such that the probability of at least one success in n trials is more than $\frac{9}{10}$.

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Solution

(i) Probability = Probability that 1st 6 trials are failures

$$= \left(\frac{9}{10}\right)^6 = 0.53144 = 0.531 \text{ (3sf)}$$

(ii) Probability = Probability that there are 2 successes in the 1st 5 trials, and then a success on the 6th trial

$$= \binom{5}{2} \left(\frac{1}{10}\right)^2 \left(\frac{9}{10}\right)^3 \times \frac{1}{10} = \frac{9^3}{10^5} = 0.00729$$

(iii) Probability of at least one success in n trials

$$= 1 - \text{Prob}(\text{no successes in } n \text{ trials})$$

$$= 1 - \left(\frac{9}{10}\right)^n$$

$$\text{Then } 1 - \left(\frac{9}{10}\right)^n = \frac{9}{10}$$

$$\Rightarrow \left(\frac{9}{10}\right)^n = \frac{1}{10}$$

$$\Rightarrow n \ln(0.9) = \ln(0.1)$$

$$\Rightarrow n = 21.854$$

So n needs to be 22.

(2) Repeated independent trials of an experiment are carried out. On each trial the probability of success is $\frac{1}{5}$.

(i) Find the probability that the 1st success occurs after the 5th trial.

(ii) Find the probability that the 2nd success occurs on the 10th trial.

(2) Repeated independent trials of an experiment are carried out. On each trial the probability of success is $\frac{1}{5}$.

(i) Find the probability that the 1st success occurs after the 5th trial.

(ii) Find the probability that the 2nd success occurs on the 10th trial.

Solution

(i) Probability = Probability that 1st 5 trials are failures

$$= \left(\frac{4}{5}\right)^5 = 0.32768 \text{ (5sf) or } 0.328 \text{ (3sf)}$$

(ii) Probability = Probability that there is exactly one success in the 1st 9 trials, and then a success on the 10th trial

$$= \binom{9}{1} \left(\frac{1}{5}\right)^1 \left(\frac{4}{5}\right)^8 \times \frac{1}{5} = 0.060398 \text{ (5sf) or } 0.0604 \text{ (3sf)}$$