Game Theory – Q2 [12 marks](28/5/21)

Exam Boards

OCR : D (Year 1)

MEI: -

AQA: D (Year 1)

Edx: D2 (Year 2)

A zero-sum game is given by the following pay-off matrix (from player 1's point of view). Confirm that there is no stable solution, and find the optimal mixed strategy for each player, and their expected pay-offs.

Player 2: Player 1	A	B
A	2	3
B	4	-1

[12 marks]

A zero-sum game is given by the following pay-off matrix (from player 1's point of view). Confirm that there is no stable solution, and find the optimal mixed strategy for each player, and their expected pay-offs.

Player 2: Player 1	A	B
A	2	3
B	4	-1

[12 marks]

Solution

Player 2: Player 1	A	B	row min.
A	2	3	(2)
B	4	-1	-1
col. max.	4	(3)	

[2 marks]

As the max. of the row minima doesn't equal the min. of the col. maxima, there is no stable solution. [1 mark]

Let p be the probability that player 1 chooses option A.

Then the expected pay-off for player 1 if player 2 chooses A is:

$2p + 4(1 - p) = 4 - 2p$ [1 mark]

and if player 2 chooses B it is:

$3p + (-1)(1 - p) = 4p - 1$ [1 mark]

The optimal value of p occurs when $min(4-2p, 4p-1)$ is maximised, and this occurs at the intersection of the lines $y = 4 - 2p$ and $y = 4p - 1$ [1 mark]

ie when $4 - 2p = 4p - 1 \Rightarrow 6p = 5; p = \frac{5}{6}$ [1 mark]

and the expected pay-off for player 1 is $4 - 2\left(\frac{5}{6}\right) = \frac{7}{3}$ [1 mark]

Similarly, let q be the probability that player 2 chooses option A.

Then the expected pay-off for player 2 if player 1 chooses A is:

$(-2)q + (-3)(1-q) = q - 3$ [1 mark]

and if player 1 chooses B it is:

$(-4)q + (1)(1-q) = 1 - 5q$ [1 mark]

The optimal value of q occurs when $min(q-3, 1-5q)$ is maximised, and this occurs at the intersection of the lines $y = q - 3$ and $y = 1 - 5q$

ie when $q - 3 = 1 - 5q \Rightarrow 6q = 4; q = \frac{2}{3}$ [1 mark]

and the expected pay-off for player 2 is $\frac{2}{3} - 3 = -\frac{7}{3}$ [1 mark]

[$-1 \times$ player 1's expected pay-off, as would be expected]