Friction – Q3 [9 marks](3/6/21)

Exam Boards

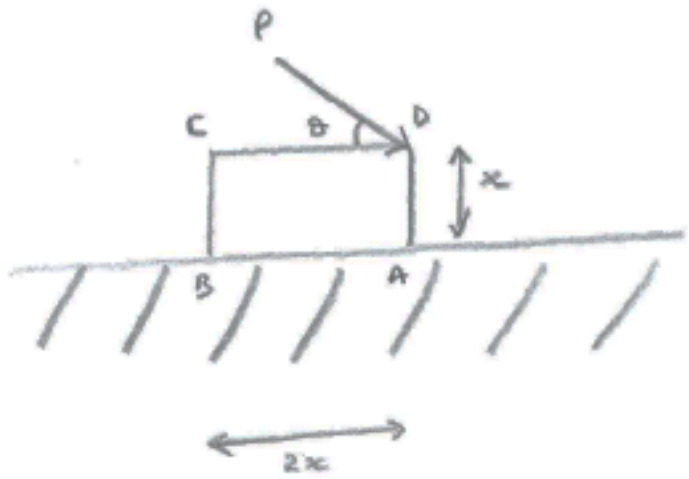
OCR : Mechanics (Year 2)

MEI: Mechanics a

AQA: Mechanics (Year 2)

Edx: Mechanics 2 (Year 2)

A uniform block of mass m rests on a table, and a force P is applied at D, as shown in the diagram. The block has length $2x$ and height x. The coefficient of friction between the block and the table is μ.



(i) If the block is on the point of sliding, find an expression for P.

[3 marks]

(ii) If instead the block is on the point of toppling, find an expression for P. [3 marks]

(iii) If the block is to topple before it slides, find a condition on μ.

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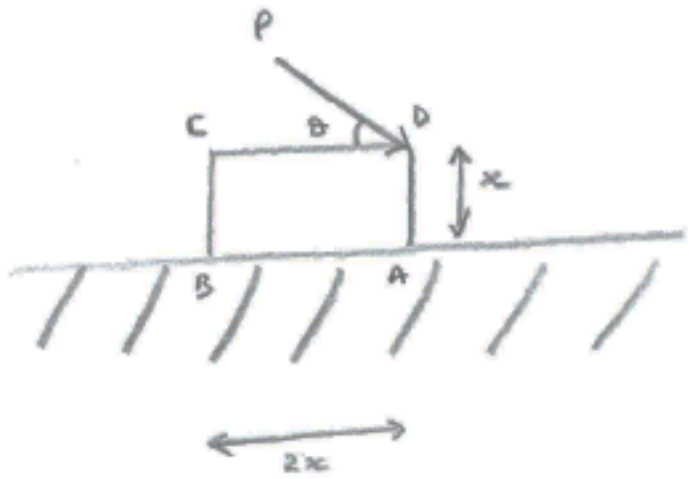

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[3 marks]

Solution

(i) The normal reaction, $R = mg + Psin\theta$

The frictional force $= \mu(mg + Psin\theta)$ [1 mark]

Hence, at the point of sliding, $\mu(mg + Psin\theta) = Pcos\theta$, [1 mark]

so that $P(cos\theta - \mu sin\theta) = \mu mg$

and $P = \frac{\mu mg}{cos\theta - \mu sin\theta}$ [1 mark]

(ii) If the block is on the point of toppling, it will be about A, and the only reaction on the block will be at A. [This will be a combination of a normal reaction and friction.] [1 mark]

As the block is uniform, its weight will act at a distance x from AD, and so, taking moments about A,

$(mg)x = (Pcos\theta)x$ [1 mark]

[the normal reaction and friction contribute nothing, as they act at A]

Hence $P = \frac{mg}{cos\theta}$ [1 mark]

(iii) At the critical position where the block is about to both slide and topple,

$P = \frac{\mu mg}{cos\theta - \mu sin\theta} = \frac{mg}{cos\theta}$ [1 mark]

so that $\mu cos\theta = cos\theta - \mu sin\theta$;

$\mu(cos\theta + sin\theta) = cos\theta$

and $\mu = \frac{cos\theta}{cos\theta + sin\theta} = \frac{1}{1+tan\theta}$ [1 mark]

So, if the block is to topple before it slides, we require

$\mu > \frac{1}{1+tan\theta}$ [ie making the frictional force greater] [1 mark]

[reasonableness check: if $\theta = 45°$, then $\mu > 0.5$; also, if θ is reduced to 30°, we would expect a higher value of μ to be necessary, in order for toppling to occur first (since the block is now more prone to slide than topple), and the condition gives $\mu > \frac{1}{1+\frac{1}{\sqrt{3}}} = 0.634$]