Friction – Exercises (with Sol'ns) (11 pages; 8/2/26)

(1) A sledge with a child onboard is being pulled along on level ground, at a constant speed, by means of a rope inclined at 30° to the horizontal. The sledge and child together have a mass of $100kg$. The coefficient of friction between the sledge and the ground is $\frac{1}{10}$. Assuming that $g = 10$, find the tension in the rope.

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Solution

Let T be the tension, and let R be the normal reaction of the ground on the sledge. Then, applying N2L vertically:

$R + Tsin30° = 100g$

Applying N2L horizontally, $Tcos30° = \mu R$

Hence $T\left(\frac{\sqrt{3}}{2}\right) = \frac{1}{10}\left(1000 - \frac{T}{2}\right)$,

so that $T\left(\frac{\sqrt{3}}{2} + \frac{1}{20}\right) = 100$

and $T = 109\ N\ (3sf)$

(2) A block rests on a slope which is angled at $\theta°$ to the horizontal. The coefficient of friction between the surface of the slope and the block is $tan\ \alpha$. P_1 is the horizontal force that needs to be applied to the block to stop it from slipping down the slope, whilst P_2 is the greatest horizontal force that can be applied without the block slipping up the slope.

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(ii) Explain what happens when $\theta < \alpha$

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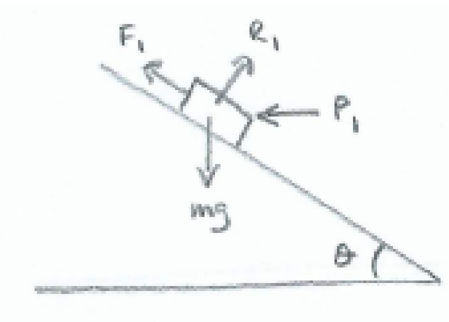
Solution

(i) As friction acts to oppose attempted motion, the frictional force acts up the slope in the first case, and down in the second.



In the first case, resolving forces perpendicular to the slope,

$N2L \Rightarrow R_1 = mgcos\theta + P_1 sin\theta$

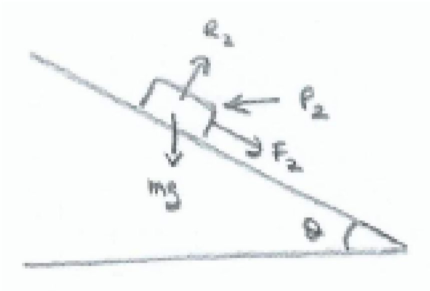
Limiting friction $\Rightarrow F_1 = tan\alpha R_1$

Resolving forces along the slope,

$N2L \Rightarrow F_1 + P_1 cos\theta = mgsin\theta$

Hence $tan\alpha(mgcos\theta + P_1 sin\theta) + P_1 cos\theta = mgsin\theta$

and so $P_1(cos\theta + tan\alpha sin\theta) = mg(sin\theta - tan\alpha cos\theta)$ (1)



In the second case, $R_2 = mgcos\theta + P_2 sin\theta$, $F_2 = tan\alpha R_2$

and $F_2 + mgsin\theta = P_2 cos\theta$

Hence $tan\alpha(mgcos\theta + P_2 sin\theta) + mgsin\theta = P_2 cos\theta$

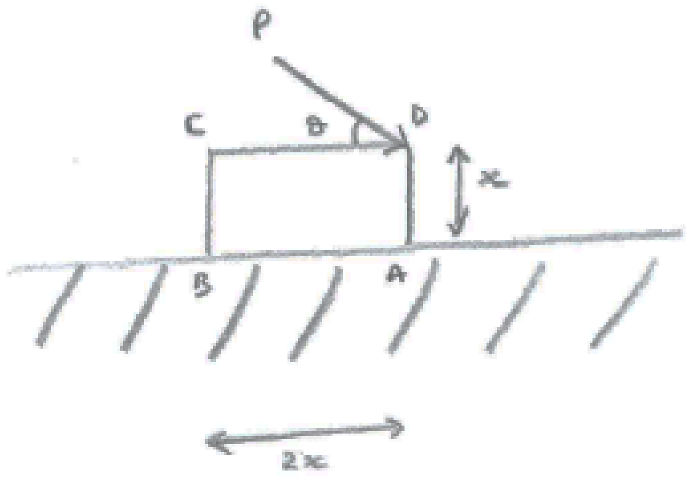
and so $P_2(cos\theta - tan\alpha sin\theta) = mg(sin\theta + tan\alpha cos\theta)$ (2)

Then (1) & (2) $\Rightarrow \frac{P_2}{P_1} = \frac{(sin\theta+tan\alpha cos\theta)(cos\theta+tan\alpha sin\theta)}{(cos\theta-tan\alpha sin\theta)(sin\theta-tan\alpha cos\theta)}$

$= \frac{(tan\theta+tan\alpha)(1+tan\alpha tan\theta)}{(1-tan\alpha tan\theta)(tan\theta-tan\alpha)} = \frac{\tan(\theta+\alpha)}{\tan(\theta-\alpha)}$

(ii) $tan\theta$ is the minimum value of μ required for the block to rest on the slope without slipping. Hence, when $\theta < \alpha$, so that $tan\theta < tan\alpha = \mu$, no horizontal force is needed.

(3) A uniform block of mass m rests on a table, and a force P is applied at D, as shown in the diagram. The block has length $2x$ and height x. The coefficient of friction between the block and the table is μ.



(i) If the block is on the point of sliding, find an expression for P.

(ii) If instead the block is on the point of toppling, find an expression for P.

(iii) If the block is to topple before it slides, find a condition on μ.

Solution

(i) The normal reaction, $R = mg + Psin\theta$

The frictional force $= \mu(mg + Psin\theta)$

Hence, at the point of sliding, $\mu(mg + Psin\theta) = Pcos\theta$,

so that $P(cos\theta - \mu sin\theta) = \mu mg$

and $P = \frac{\mu mg}{cos\theta - \mu sin\theta}$

(ii) If the block is on the point of toppling, it will be about A, and the only reaction on the block will be at A. [This will be a combination of a normal reaction and friction.]

As the block is uniform, its weight will act at a distance x from AD, and so, taking moments about A,

$(mg)x = (Pcos\theta)x$

[the normal reaction and friction contribute nothing, as they act at A]

Hence $P = \frac{mg}{cos\theta}$

(iii) At the critical position where the block is about to both slide and topple,

$P = \frac{\mu mg}{cos\theta - \mu sin\theta} = \frac{mg}{cos\theta}$

so that $\mu cos\theta = cos\theta - \mu sin\theta$;

$\mu(cos\theta + sin\theta) = cos\theta$

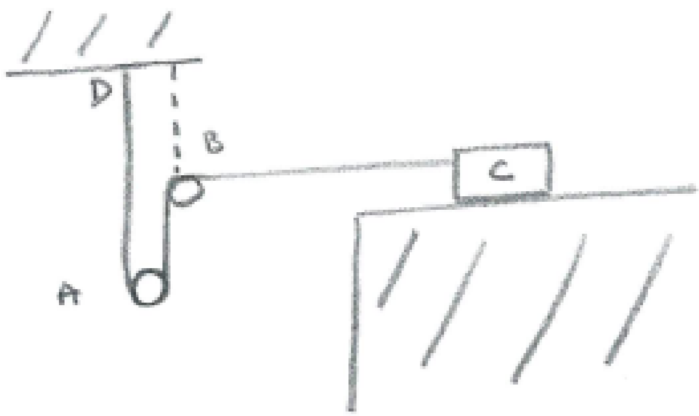
and $\mu = \frac{cos\theta}{cos\theta + sin\theta} = \frac{1}{1 + tan\theta}$

So, if the block is to topple before it slides, we require

$\mu > \frac{1}{1+tan\theta}$ [ie making the frictional force greater]

[Reasonableness check: if $\theta = 45°$, then $\mu > 0.5$; also, if θ is reduced to 30°, we would expect a higher value of μ to be necessary, in order for toppling to occur first (since the block is now more prone to slide than topple), and the condition gives $\mu > \frac{1}{1+\frac{1}{\sqrt{3}}} = 0.634$]

(4) Referring to the diagram, A is a smooth pulley of mass 2 kg, which can move up and down; B is a smooth, fixed pulley, and C is a block of mass 1kg, which is initially held at rest on a table. A light inextensible rope is fixed at D, and leads to C, via the two pulleys.



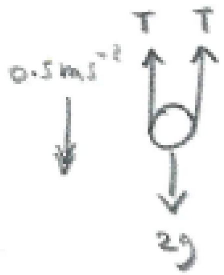
C is now released and accelerates at 2 ms^{-1}. Find the coefficient of friction, μ between C and the table.

Solution

First of all, the tension (T, say) is the same throughout the rope (since the rope is light and the pulleys are smooth - see note (i) below).

Also, because A falls by half the distance that C moves, its acceleration is also half that of C - see note (ii) below).

[The fact that the rope is inextensible ensures that the rope (as a whole) and the block move the same distances.]



From the force diagram for A,

N2L $\Rightarrow 2g - 2T = (2)(1) \Rightarrow T = g - 1$ [1]

From the force diagram for B,

N2L $\Rightarrow T - \mu R = (1)(2)$

Also, vertical equilibrium $\Rightarrow R = g$,

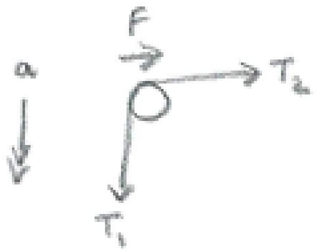
so that $T - \mu g = 2$ [2]

Then [1]&[2] $\Rightarrow \mu g = g - 3$,

so that $\mu = 1 - \frac{3}{9.8} = 0.694$ (3sf)

Notes

(i) Referring to the force diagram for the rope around B, for example:



Suppose that the rope has mass m, that the pulley exerts a frictional force F, and that the rope experiences forces T_1 and T_2 at its ends.

If the rope has acceleration a,

then $T_1 - T_2 - F = ma$

If $F = 0$ (ie if the pulley is smooth), and $m \approx 0$ (ie the rope is 'light', and so has negligible mass), then $T_1 - T_2 \approx 0$, and so the tensions are approximately equal.

(ii) From the suvat equation, if $u = 0$, then $s = \frac{1}{2}at^2$, so that the acceleration is proportional to the distance, for a given t.