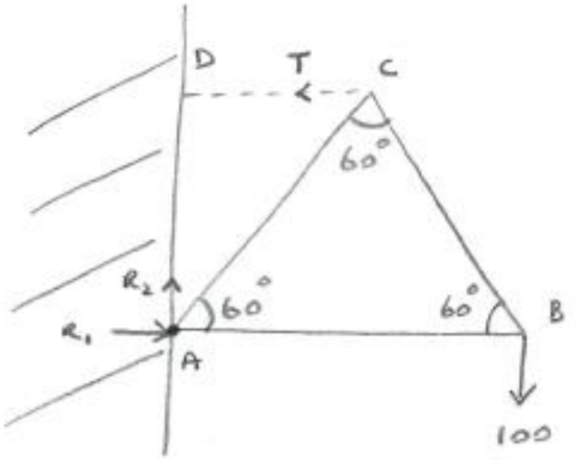
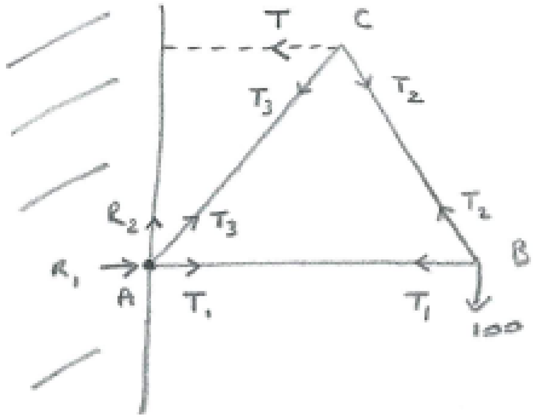
Frameworks (4 pages; 15/9/13)

The diagram below shows a framework ABC of light, freely-jointed rods in the shape of an equilateral triangle. The framework is freely hinged to the wall at A. C is connected to the wall at D by a light string, which is taut. AB and CD are horizontal. A weight of 100 N hangs from B. The problem is to find the tension in the string (T) and the components of the reaction of the wall on the framework (R_1 and R_2), as well as the tensions or compressions in the rods.



For framework questions, it is not usually worthwhile drawing separate force diagrams for each object (ie the joints A, B and C here). The above diagram shows the external forces on the framework, with the internal forces added in below.



The convention for the internal forces is for inward-pointing arrows to denote a tension. The arrows are in effect the forces exerted by the rod on the joint (then, by Newton's 3rd law, the forces exerted on the rod by the joints are directed outwards; ie the rod is being pulled at each end, and is therefore under tension).

To tackle framework questions

(i) Obtain equations for the external forces (generally weights and reactions of surfaces) by:

(a) resolving vertically & horizontally

(b) taking moments about a convenient point

Note that, in general, a surface can exert a force on a framework in any direction – and this force can be resolved into horizontal and vertical components (R_1 and R_2 in the diagram); ie the normal reaction and frictional forces, respectively, in this case.

(ii) Draw in the internal forces – ie the tensions or compressions in the rods. Assume initially that they are all tensions. A negative answer will then imply a compression rather than a tension.

(iii) Set up equations for each joint by resolving forces in perpendicular directions (eg along and perpendicular to a particular rod - whatever is most convenient). Aim to start with a joint where there are only two unknown forces, so that they can be determined from the two equations.

Example

Applying this procedure to the example above:

Resolving the external forces vertically & horizontally gives:

$\boldsymbol{R_2}$ = 100 and $\boldsymbol{R_1}$ = T

Taking moments about A gives:

T d cos $\mathbf{30^0}$ = 100 d , where d is the length of the sides of the equilateral triangle,

so that T = $\frac{\mathbf{100}}{\left(\frac{\sqrt{3}}{2}\right)} = \frac{\mathbf{200}}{\sqrt{3}} = \frac{\mathbf{200}\sqrt{3}}{\mathbf{3}}$

Thus T = $\frac{\mathbf{200}\sqrt{3}}{\mathbf{3}}$ = 115 N , $\boldsymbol{R_1}$ = $\frac{\mathbf{200}\sqrt{3}}{\mathbf{3}}$ = 115N and $\boldsymbol{R_2}$ = 100 N

We now need to find the internal forces. In this example, there are only 2 unknown forces at each joint (and 2 equations that can be used to find them), so that it doesn't matter where we start.

Starting at C, for example, and resolving forces vertically & horizontally gives:

$\boldsymbol{T_3}\cos \mathbf{30^0} + \boldsymbol{T_2}\cos \mathbf{30^0} = 0$ (1)

and T + $\boldsymbol{T_3}\cos \mathbf{60^0} = \boldsymbol{T_2} \cos \mathbf{60^0}$ (2)

(1) ⇒ $\boldsymbol{T_3} = -\boldsymbol{T_2}$

Then (2) ⇒ $\quad T = 2\,\mathbf{T_2}.\frac{1}{2} = \boldsymbol{T_2}$, and so $\mathbf{T_2} = \frac{200\sqrt{3}}{3}$ N

ie there is a tension of $\frac{200\sqrt{3}}{3} = 115$N in BC

and a compression of $\frac{200\sqrt{3}}{3} = 115$N in AC

As there are only three forces at B, compared to the four at A, resolving forces vertically & horizontally at B gives:

$\boldsymbol{T_2}\cos \mathbf{30^0} = 100$ and $\boldsymbol{T_1} + \boldsymbol{T_2}\cos \mathbf{60^0} = 0$

so that $\boldsymbol{T_1} = -\boldsymbol{T_2}\,.\frac{1}{2} = -\frac{200}{\sqrt{3}}\,.\frac{1}{2} = -\frac{100\sqrt{3}}{3}$

[the first equation also provides a check on $\boldsymbol{T_2}$]

ie there is a compression of $\frac{100\sqrt{3}}{3} = 57.7$N in AB