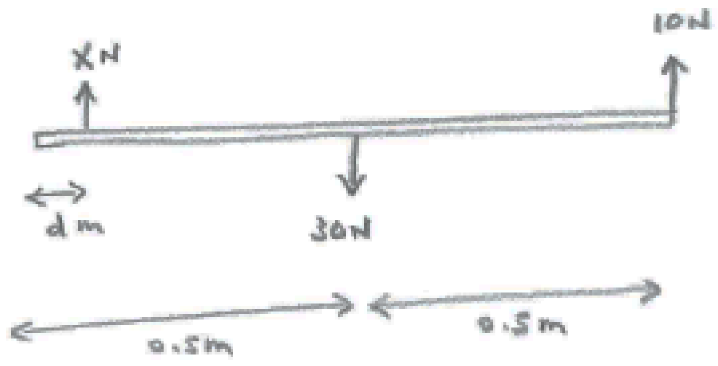
Forces – Q2 [Practice/M] (2/6/21)

Vertical forces of X, 30 and 10 N are applied to a light rod of length 1 m, as shown in the diagram. The force of X N is applied at a distance of d m from the left-hand end, and the force of 30N is applied at the mid-point of the rod.



(a) What values must X and d have in order for the rod to be in equilbrium?

(b) The force of X N is removed, and the forces of 30N and 10N are to be replaced with a single force having the same effect as these two forces. What is the size and line of action of this single force?

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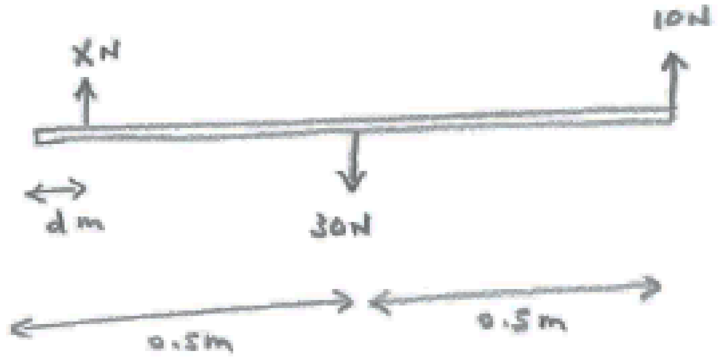

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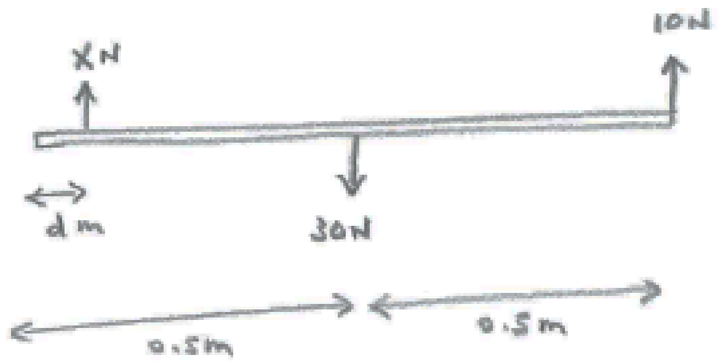
Solution



(a) Vertical equilbrium $\Rightarrow X + 10 = 30 \Rightarrow X = 20$

Taking moments about the right-hand end (for example):

$30(0.5) - 20(1 - d) = 0 \Rightarrow -5 + 20d = 0 \Rightarrow d = 0.25$

[Whenever the forces are balanced, the total moment will be the same about any point; eg taking moments about the mid-point instead:

$10(0.5) - 20(0.5 - d) = 0 \Rightarrow -5 + 20d = 0$, as before

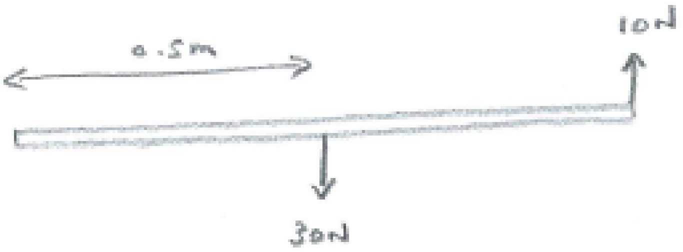
or, about the point where X is applied:

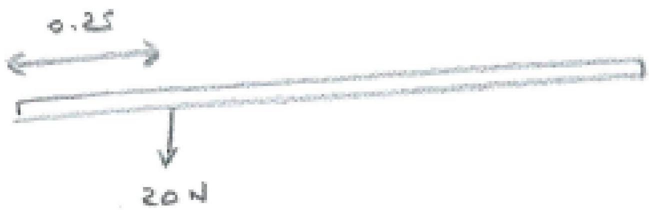
$10(1 - d) - 30(0.5 - d) = 0 \Rightarrow -5 + 20d = 0$]

(b) From (a), X counteracts the effect of the other two forces to give equilibrium.

Now a force of 20 N acting at the same position as X, but in the opposite direction, will also be counteracted by X. Hence it follows that this force is equivalent to the forces of 30 N and 10 N.

Thus the two systems shown below are equivalent.





Alternative method:

The single equivalent force must be of magnitude 20 N, acting downwards (this being the net effect of the two forces). Suppose that it acts at a distance d from the left-hand end.

Then we require the moment of this force about the left-hand end (say) to equal the net moment of the two forces.

So $-20d = 10(1) - 30(0.5) = -5 \Rightarrow d = 0.25$

(Note: the single force could not act beyond the left-hand end, as that would give rise to a positive moment, which could not be equated to -5)